Visualizing Harmonic Space

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December 2024

Contents

1 Harmonic Proximity

When discussing chords and their relationships to one another, various conventions are used to emphasize different properties of such relationships. The Roman numeral system is used to analyze chords within the context of a key center, where a I chord is the tonic, and every other chord relates to it in some way. The Roman numeral system can be seen as the chordal analog to the Movable-Do Solfège system of note naming based on a diatonic key. Another popular way of discussing note-to-note relationships is to describe their relationship as an interval. A typical convention is to describe their generic interval - *second, third, fourth, fifth, etc.* - as well as a descriptor to talk about their size - *major, minor, diminished, augmented.* This interval system is a *discrete* system, as opposed to the contextual Movable-Do system. The interval system can be used in unison with the Movable-Do system, and even highlights the fact that, for instance, the interval from *Do* to *Mi* is the same as the interval from *Fa* to *La* and *Sol* to *Ti*: a major third. By using the interval system, we can see that there are intervals that function differently in the context-bound system, but their inherent properties are the same when factoring out transposition. In set theory, pitch-class intervals and interval classes are used ubiquitously to show how composers interact with specific intervals and the properties of those intervals.

When analyzing chords, discrete harmonic analysis systems can prove to be more useful than Roman numeral systems in music that goes beyond even complex diatonic convention. There are many examples of music that possess harmonic novelties in which a context-bound system like Roman numeral analysis do not suffice for explaining *why* their progressions function as such. Transformational systems express properties of voice leading through mathematical group theory, creating a set of particular transformations that map any triad onto another triad. Most notably, the work of Hugo Riemann inspired theorists such as Richard Cohn, David Lewin, Brian Hyer, Robert Cook, and David Kopp to develop and contribute to the Neo-Riemannian triadic transformation system. This system is valuable to showcase some of the peculiarities of certain progressions that the Roman numeral system lacks, but with a reliance on solely major and minor triads, as well as a need for the transformational set to adhere to the laws of mathematical group theory, these methods also do not highlight the particular properties of such interesting progressions in a way that explains why they are so novel. The following section explains how progressions and adjacent chords can be analyzed through harmonic space, unveiling a method for observing harmonic distance. The interaction of harmonic distance is a valuable property when analyzing harmonic progressions. Analyzing chords through mapping them onto the circle of fifths reveals many unique properties of harmonic progressions that are otherwise unnoticed or unclear when using alternative methods.

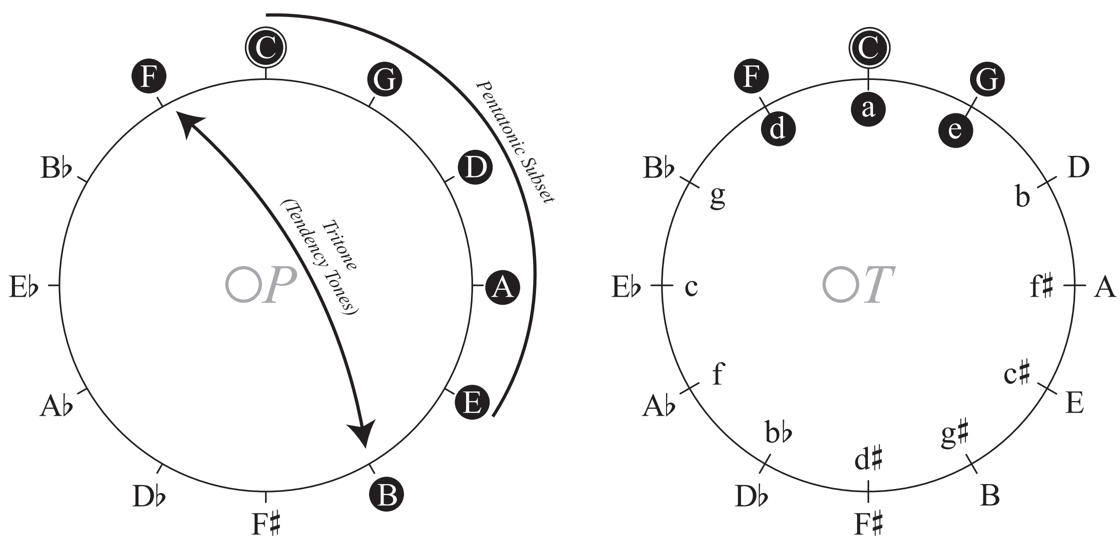


Figure 1: The notes and triads of the C major diatonic set mapped onto a Circle of Fifths using pitches (◯P), a the Circle of Fifths using major and minor triads (◯T).

1.1 The Circle of Fifths

When qualifying harmonic motion, it becomes evident that certain triadic relationships are more or less closely related to one another, either by voice leading or by harmonic proximity. The concept of harmonic proximity is derived from the measurement of distance between two or more chords on the circle of fifths. Figure 1 shows two different ways of visualizing the circle of fifths: A Pitch Circle ◯P, where each space moving clockwise is a perfect fifth (T_7) higher than the previous space, and a Triadic Circle ◯T, where major triads and their relative minor triads are mapped on two overlapping circles. ◯T aligns with the traditional circle of fifths which is typically used to show relative major and minor keys. In Figure 1, all of the notes in the C major diatonic scale are mapped onto ◯P, and all of the major and minor triads are mapped onto ◯T. The most notable observation about both of these mappings is that the diatonic major scale maps onto both ◯P and ◯T in such a way that every note and triad is adjacent to one another with zero outliers and zero gaps. Moreover, the two notes of furthest distance on ◯P are the fourth and seventh scale degrees which makeup the tritone: the most unstable interval in the diatonic pitch set. The notes of the tritone interval are the most unstable *because* they are the furthest in distance from each other on ◯P. The set of triads in the C major diatonic set are similar to those in the B♭ diatonic set *because* C and B♭ are relatively close on ◯T. Such organization of the diatonic set means that it is possible to visualize and measure the harmonic proximity of triads, harmonic centers, and even notes on these circles. In particular, measuring the proximity of notes on the circle of fifths instead of by a more typical approach of using semitones becomes an important factor in how the motion of different triadic members impacts harmonic proximity. It is clear that a link exists between the salient properties of harmony and the mapping of triads and pitches on the circle of fifths.

1.2 Visualizing Harmonic Proximity

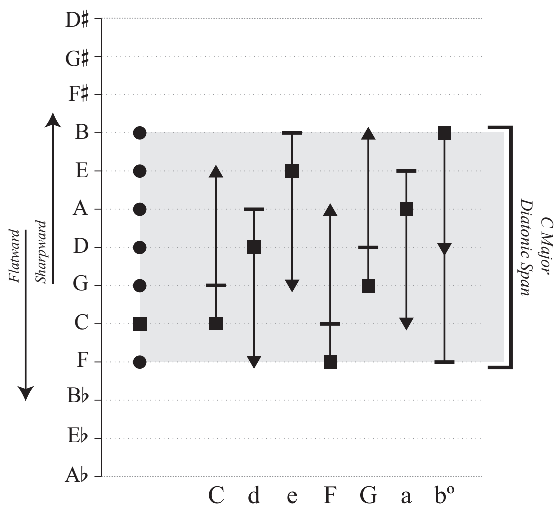


Figure 2: All of the notes and triads in the C major diatonic set mapped onto a a graph generated by ◯P on the vertical axis.

The orientation and design of the ◯P and ◯T make it very difficult to map a full harmonic progression as there is no way to express temporal order on the circle. Figure 2 shows ◯P oriented vertically where the previous clockwise - or sharpward - direction is now upward, and counterclockwise - flatward - is downward. This graph will henceforth be called the *Harmonic Proximity Graph.* Figure 2 maps all of the pitches of the C major scale as black circles, showing the same adjacency from ◯P. Doing so unlocks a visualization of the vertical span of the diatonic scale which takes up 6 vertical steps on the graph. To the right of the black dots is every possible triad within the C major diatonic set. The square is the root of the triad, the horizontal line is the fifth, and the triangle is the third. The triangle, being the symbol for the third of the triad, also makes it easy to decipher the difference between major triads with triangles pointing upward, and minor triads with triangles pointing downward. It also becomes possible to map triads (and other chords) other than major or minor on this graph, as seen by the b° triad. The symmetry of the diatonic diminished triad within the shaded area of the C major diatonic span stands out as a striking visual feature highlighted by the graph as well. The fact that its span of 6 is larger than the span of 4 in the major and minor triads shows visually that it is a more chromatic vertical sonority than the others.

1.2.1 Roles of Triad Members

The mapping of triads in this manner shines an additional light on the roles of each member of the triad. On the graph, the root and the fifth of the major or minor triad will always be one step away from one another with the fifth sharpward of the root. These members will always be in lockstep with one another, as they are the stable members of major and minor triads. The third of the triad is the member that is the most unstable, and therefore is what determines whether the triad is major or minor. When mapped on the Harmonic Proximity Graph, the major third is 3 steps sharpward of the fifth, and the minor third is three steps flatward of the root. This visual organization of the Harmonic Proximity Graph directly aligns with the well-understood roles of each triad member, and directly correlates to how progressions typically move through harmonic space. When altering a chord from its diatonic quality, the *third* is the member that is altered. Therefore, the third of the triad is the most common member to be outside of the diatonic span, as it is the most easily subjected to chromatic alteration.

1.2.2 Measuring Harmonic Distance

Similar to the previously defined diatonic span, the Harmonic Proximity Graph (Figure 2) also illustrates the extent to which a chord occupies the circle of fifths. Both major and minor triads span four fifth-steps on the graph. The general harmonic distance between two triads is the distance that their spans σ move on the circle. While both major and minor triads have the same magnitude of their spans ($\sigma = 4$), this is not always the case. A more concise way of deriving the same information is to measure the distance (Δ) between center points (μ) of the spans: $\Delta\mu$. This also provides an easy way to add more complex chords into the system. Measuring the μ center point of a chord in this fashion allows for a singular spot on the graph to describe the position of the chord. This measurement also validates the concept of relative major and minor triads, where (seen on Figure 2) a major triad takes up the same span as its relative minor triad, and vice versa. Furthermore, relative major and minor triads share the same μ using this measurement. The μ measurement treats the circle of fifths as a locally-Euclidean, 1-dimensional space. This becomes helpful later on when defining a position for more complex chords.

1.3 Diatonic Spans and Popular Progressions

The Harmonic Proximity Graph makes it possible to find many salient features of chord progressions, showing how harmonic distance influences the quality of such progressions. One common four-chord progression that has seen many variations throughout the repertoire of recent popular music[1] can be expressed in Roman numerals as I - iii - vi - IV. This particular base progression is often altered

1. David Bennett, *Songs that Use the 1 3 6 4 Chord Progression*, 2018, https://www.youtube.com/watch?v=mUYgS4IP_TU.

SONG, ARTIST, AND DATE	PROMINENT CHORD LOOP
Someone Like You - Adele (2011)	I - iii - vi - IV
She Used to Be Mine - Sara Bareilles (2015)	I - V - vi - IV
I'm Not the Only One - Sam Smith (2014)	I - III - vi - IV
Creep - Radiohead (1992)	I - III - IV - iv

Table 1: The I-iii-vi-IV triadic progression and three variations.

using modal mixture techniques. Table 1 shows one example of an unaltered I - iii - vi - IV from Adele's *Someone Like You*, and three other progressions that possess various alterations. Each progression begins with a I chord moving to some sort of chord with a root of $\hat{3}$, either major or minor. The progressions then move to either a vi chord or a IV chord - both non-tonic stable chords, and then to a IV, or iv chord, preparing a cadential move back to the tonic at the beginning of the loop. By solely looking at the Roman numerals, it becomes clear *how* the progressions are different - particularly by their root motion and their major or minor qualities - but the Roman numeral system does not inherently explain the differences between the progressions' other properties, such as voice leading and harmonic distance.

2 Four Variations on a Progression

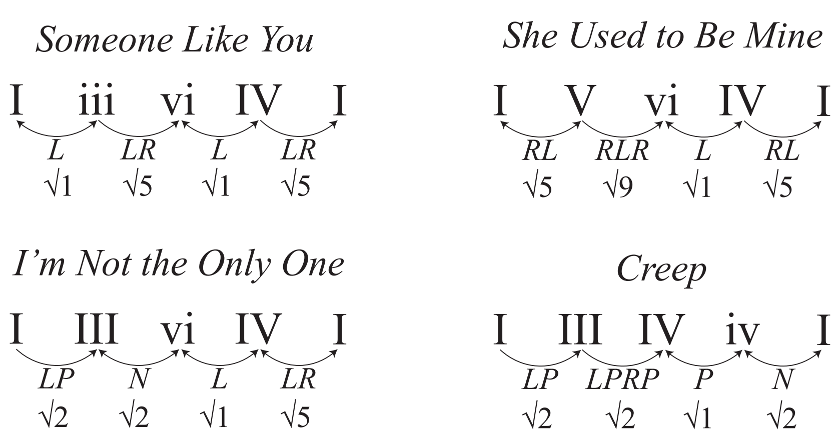


Figure 3: The four variations of I-iii-vi-IV analyzed for P,L,R,S,N transformations and voice leading proximity using the L^2 vector norm.

Figure 3 uses the system of transformations developed by Cohn, Lewin, Hyer, and Cook to show the differences between these progressions using transformation theory. The second level accounts for how many semitone moves are between adjacent triads using the L^2 vector norm. The transformation system highlights the difference between I→iii and I→III, where both possess *L* (leading-tone) transformations, but the more distant transformation from I to III needs the additional *P* (parallel) transformation. The transformations also show that our "control" progression in *Someone Like You* creates a *L*→*LR* loop, where only two separate transformations are needed to create the entire progression.

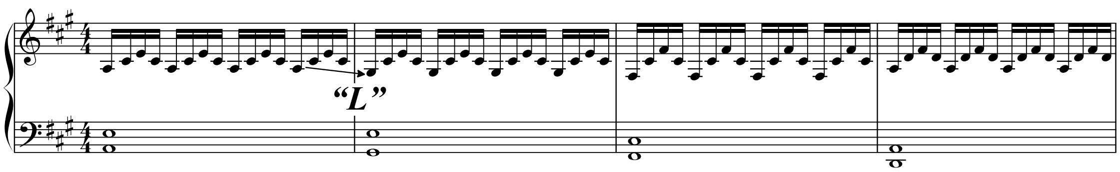


Figure 4: The opening four measures of Adele's *Someone Like You*, showing a composed *L* transformation.

It is easy to see that Adele's *Someone Like You* was conceived as a voice leading progression. The opening piano arpeggiation moves from the I chord to the iii chord using the most concise voice leading motion. Figure 4 shows how this particular voicing in the right hand is a true *L* transformation. The motion from vi to IV is also an *L* transformation, but the voicing of the chord change in Figure 4 does not follow the same parsimonious voice leading.

Figure 5 shows an altered voicing of the same progression that adheres to the Neo-Riemannian transformations that are at play. This version contains the most parsimonious voice leading between chords. Obviously, Adele did not decide to voice the opening in this way, which means that there are other properties at play other than just voice leading. Analyzing this progression for voice leading is conclusively insufficient.

Figure 5: The progression from Adele's *Someone Like You* with parsimonious voice leading.

2.0.1 Progressions on the Graph

Both the Roman numeral system and the transformation system together show more about the novelties within each progression than either system can show by itself. When these progressions

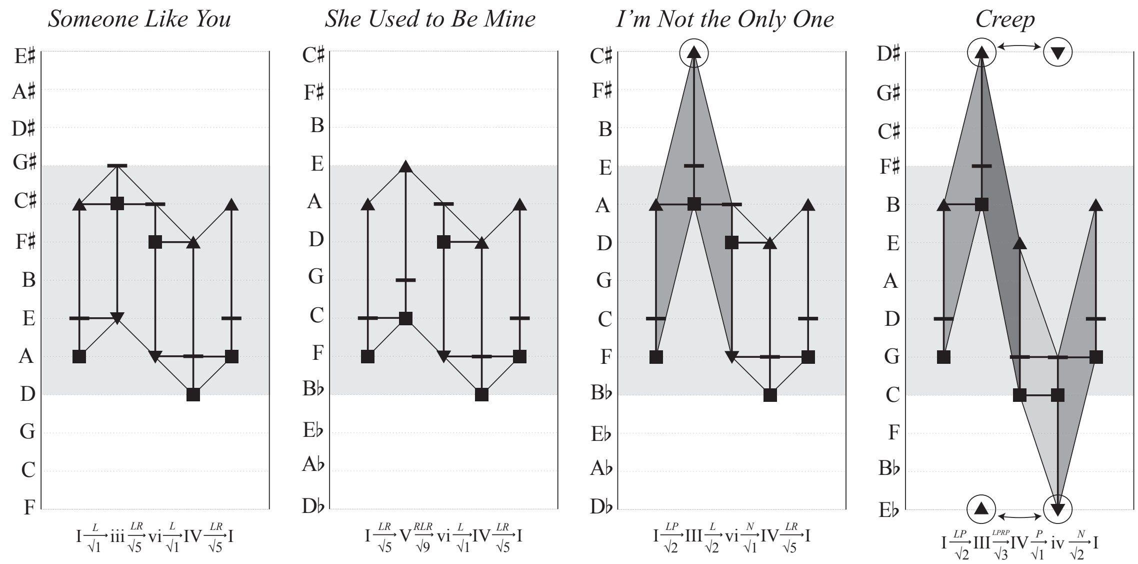


Figure 6: The four progressions mapped onto the Harmonic Proximity Graph.

are graphed on the Harmonic Proximity Graph, these novelties become even more evident. The most immediately striking aspect of Figure 6 is that there is a clear division between the first two examples and the second two: *Someone Like You* and *She Used to Be Mine* stay completely within the diatonic span of their key centers, while *I'm Not the Only One* and *Creep* briefly venture out of the key center. Both *Someone Like You* and *She Used to Be Mine* exemplify typical diatonic chord progressions in popular music. Both contain some of the strongest diatonic chords in popular music: vi, and IV, as well as a plagal-cadence figure that seals up the looping progressions. The progressions show two common approaches to vi: iii and V. Because these chords are relative to one another, the $\Delta\mu$ is identical for both of these progressions. The graph's diagonal lines in parallel connecting each adjacent triad show how the spans of each chord move in harmonic space. This is a visual representation of $\Delta\mu$. The horizontal bars connecting adjacent triads show the common tones between them. Compared to *She Used to Be Mine*, the progression from *Someone Like You* possesses more common tones throughout the loop. The visual difference between *Someone Like You* and *She Used to Be Mine* is solely a product of the small difference between the iii and V chords in each loop. The choice between the V and the iii chord subtly changes the properties of the progression, and the graph shows exactly why these progressions differ as a result of this choice.

2.0.2 Chromatic Alterations

Similar to the subtle difference between *Someone Like You* and *She Used to Be Mine*, the only difference between *Someone Like You* and *I'm Not the Only One* is the use of a major III chord

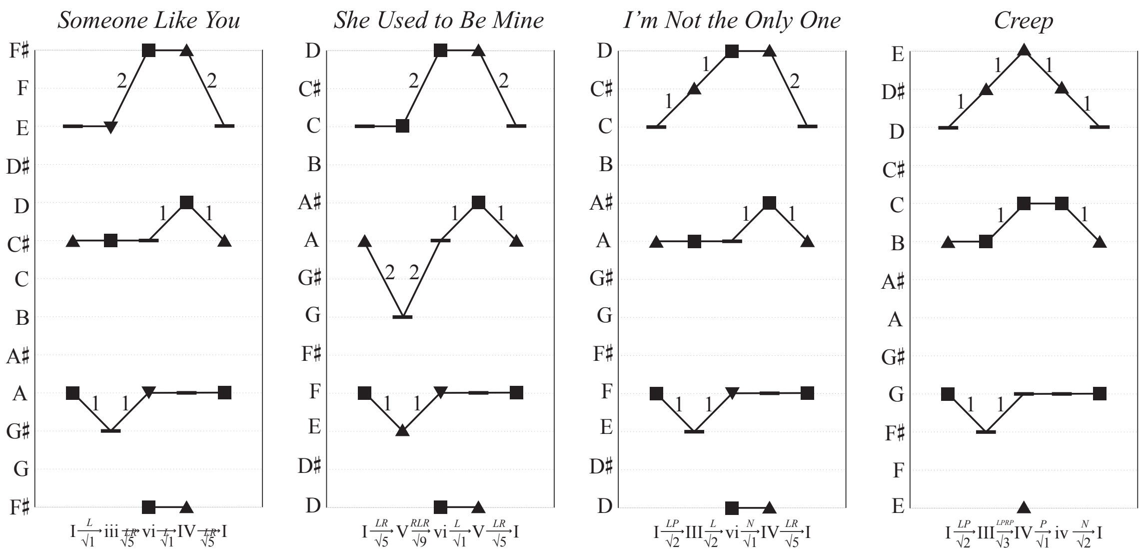


Figure 7: The four progressions on a *Voice Leading Proximity Graph* showing semitones on the vertical axis as opposed to perfect fifths.

versus a minor iii chord. When using Roman numerals, these progressions look very similar. The Neo-Riemannian transformation system highlights more of the difference between the two ($L \to LR$ versus $LP \to N$) and how the progression changes due to the change in quality of the iii chord. Graphing these progressions directly shows *why* the particular chromatic alteration changes the harmonic properties of the progression. The altered third of the III chord in *I'm Not the Only One* moves the harmony 4 steps sharpward ($\Delta\mu = \sharp 4$) instead of the ♯1 in *Someone Like You*. The circled triangle in Figure 6 is visually very far away from the rest of the progression, and is three steps sharpward from the diatonic span. This results in a uniquely chromatic quality within this section of the progression, and could be a reason why this progression stands out sonically when compared to *Someone Like You*.

2.1 Variants of Similar Harmonic Motion

Immediately following the $\Delta\mu$ of ♯4 in *I'm Not the Only One* is an equally flatward fall by four steps (♭4). This motion looks harmonically quite similar to the ♯4 on the graph, as both move by four steps in opposite directions, but there is an important difference. The progression from F→A is mode-preserving; a characteristic which dictates that all of the members of the former triad move the same distance on the graph - ♯4 - to reach the latter triad. The following progression from A→d is mode-reversing, where the triad members do not move equally from the former to the latter triad.

Figure 8 shows the ♯4 (as in the F→A) and the retrograde of the progression A→d - d→A -

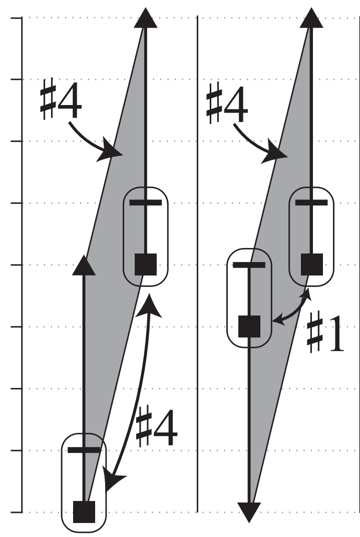


Figure 8: A comparison of two non-identical progressions that possess identical $\Delta\mu$ values

in order to more clearly see the difference between the two progressions. The way in which major and minor triads are mapped onto the circle of fifths places the root and the fifth of the triads adjacent to one another in both instances. The third - being the differentiating member between major and minor triads - is positioned three steps away from the root and fifth. The concentration of the root and fifth creates a higher sense of harmonic position than the third. In Figure 8, the second progression possesses a much smaller distance between respective root-fifth pairs than the first progression. This discrepancy calls for an additional measurement in order to differentiate between chords that possess identical spans (in this case, relative major and minor triads). The μ center point measurement does not account for the asymmetrical distribution of members on the circle. The way to incorporate the unique positioning of members within major and minor triads into a positional measurement is to take the circular mean φ of the triad.

The circular mean (φ) of a chord's members shows the exact center point of the triad when taking into consideration the relative placement of triad members on the circle of fifths. The process for deriving φ is shown in Figure 9. For a major triad, φ is sharpward of the fifth, and for a minor triad, φ is flatward of the root. Figure 9 also shows the discrepancy between the placement of μ and φ on major and minor triads. The μ measurement equates relative chords, while the φ creates a unique position between them. Analyzing the distances between circular means ($\Delta\varphi$) of adjacent triads unveils the inherent difference between non-identical progressions that share the same $\Delta\mu$ distance.

Figure 10 shows a progression of major triads in which $\Delta\mu = 4$ (the F→A from earlier being one

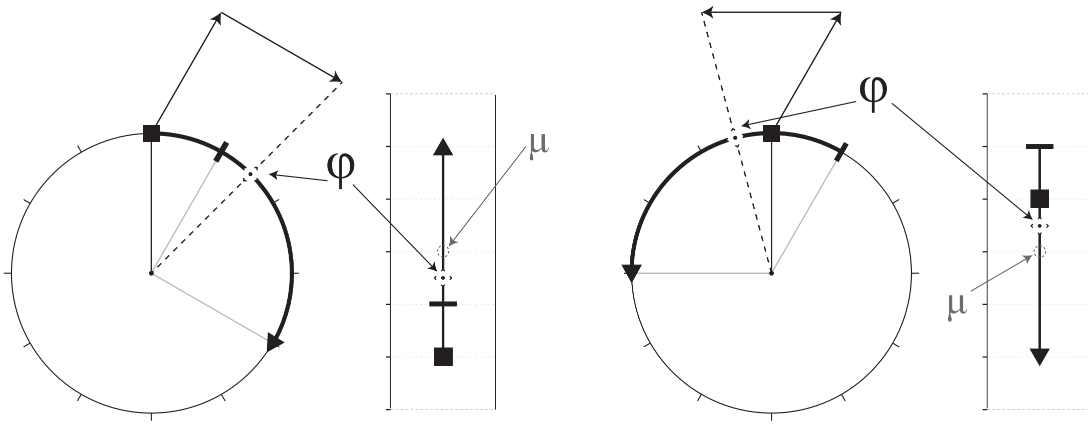


Figure 9: The center point of a major and minor triad by calculating the circular mean (φ).

example of this) accompanied by two mode-reversing progressions where $\Delta\mu = 4$ as well. The mode-preserving progression possesses identical $\Delta\mu$ and $\Delta\varphi$, while in the mode-reversing progressions, $\Delta\mu$ and $\Delta\varphi$ are unique. The progression beginning on a minor triad and moving to a major triad possesses a smaller $\Delta\varphi$ than the $\Delta\mu$, and the progression moving from major to minor possesses a $\Delta\varphi$ that is higher than the $\Delta\mu$. The interaction between $\Delta\mu$ and $\Delta\varphi$ in purely triadic music is highly consistent and predictable, giving cause to combine $\Delta\mu$ and $\Delta\varphi$ into a measurement of Triadic Harmonic Distance: ΔT. ΔT is used to measure $\Delta\mu$ and the subsequent effect that $\Delta\varphi$ has on $\Delta\mu$ in progressions that possess only major or minor triads. Equation 1 details the logic of ΔT.

$$\Delta T = \begin{cases} \Delta\mu & \text{if } \Delta\varphi = \Delta\mu \\ \Delta\mu + & \text{if } \Delta\varphi > \Delta\mu \\ \Delta\mu - & \text{if } \Delta\varphi < \Delta\mu \end{cases} \tag{1}$$

When speaking only about major and minor triads, the $\Delta\varphi$ of a given mode-reversing progression will either be one step larger or smaller than the $\Delta\mu$. For the examples in Figure 10, the ♯4– ("Lowered Sharp Four") has a $\Delta\varphi$ of 3, while the ♯4+ ("Raised Sharp Four") has a $\Delta\varphi$ of 5. The ♯4– progression has more in common with the regular ♯4 than the ♯3 progression, which is why it is more valuable to quantify the progressions by their $\Delta\mu$ first and foremost, with the respective $\Delta\varphi$ distances affecting the $\Delta\mu$ by "raising" (+) or "lowering" (–). All mode-preserving progressions between major or minor triads possess these interactions between $\Delta\mu$ and $\Delta\varphi$, therefore any

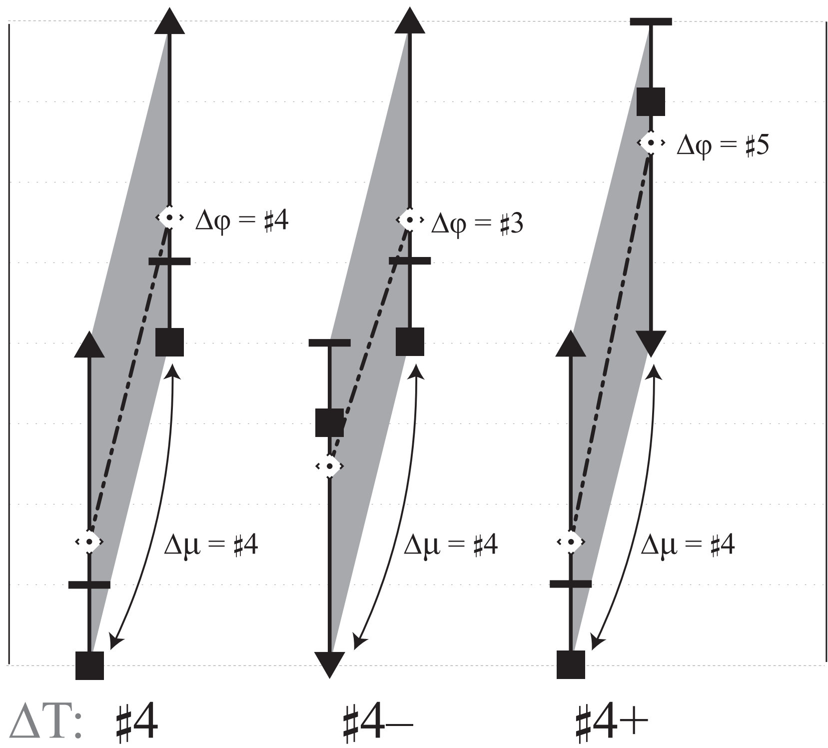


Figure 10: The difference between a mode-preserving move by four harmonic steps and mode-reversing moves by four harmonic steps.

progression between major or minor triads can be qualified by direction and distance on the circle of fifths - $\Delta\mu$- and by the agreement or disagreement of $\Delta\mu$ and $\Delta\varphi$.

As seen in Figure 6, in *I'm Not the Only One*, the progression from F→A is ♯4, and the following progression from A→d is ♭4– as the $\Delta\varphi$ lowers the effect of the ♭4. This relationship is similar to that of *She Used to Be Mine* and *Someone Like You*, where the progression from I→iii (or I→V) is ♯1+ and the following progression from iii→vi (V→vi) is ♭1. Here, the ♭1 is a harmonically lowered version of the previous ♯1+, creating a similar rise in tension followed by a resolution.

Jason Yust derives the exact same center point between major and minor triads via the circular mean of the triad[2]. In fact, the Y axis of Yust's toroidal *Tonnetz* is identical to the Y axis of the Harmonic Proximity Graph. Figure 11 shows how each member of the C major and A minor triads, as well as their center points on Yust's torus can be directly translated to their representation on the Harmonic Proximity Graph.

2. Jason Yust, "Schubert's Harmonic Language and Fourier Phase Space," *Journal of Music Theory* 59, no. 1 (2015): 121–181

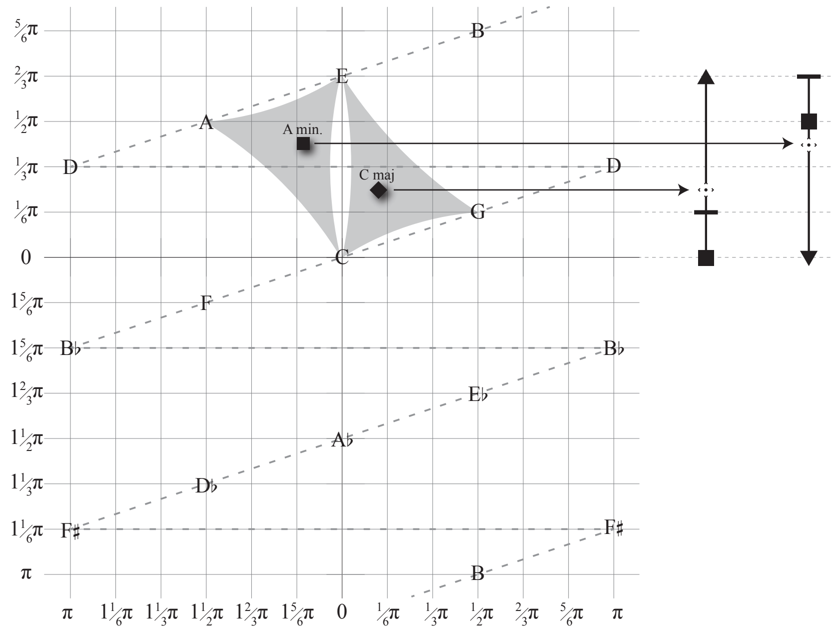


Figure 11: Yust's Tonnetz, showing the center point of two relative triads: C major and A minor, as well as the exact coordination of φ between the torus and the Harmonic Proximity Graph.

2.1.1 Root Motion

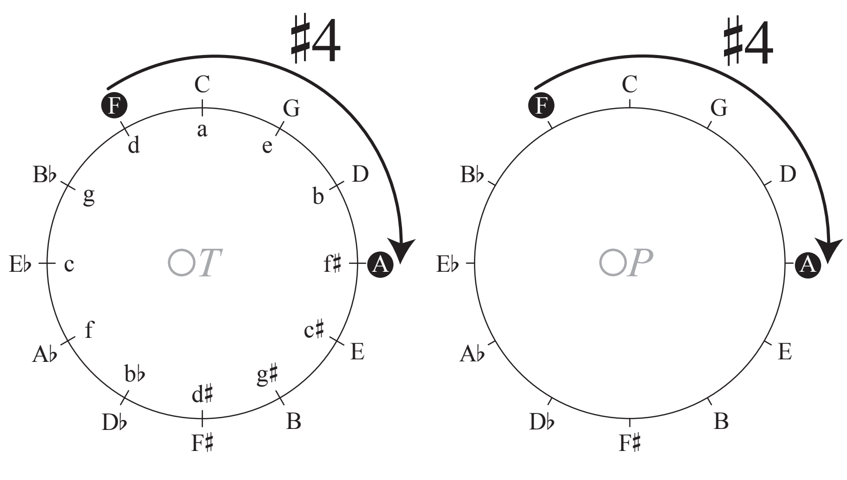


Figure 12: F→A mapped onto $\bigcirc P$ and $\bigcirc T$.

An alternative way to understand the difference between non-identical progressions with the

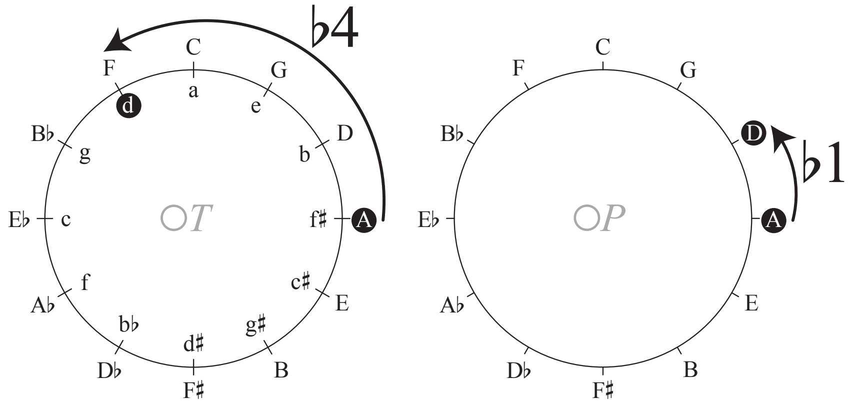


Figure 13: A→d mapped onto ◯P and ◯T.

same harmonic distance is by comparing the harmonic distance ($\Delta\mu$) to the distance of the root ($\Delta\rho$). Figure 12 shows the progression F→A (♯4) mapped onto ◯T and ◯P. Because the progression does not change quality, the distance on ◯T and ◯P are identical. This is evident in Figure 10 as well, where the root moves in parallel with the harmonic distance. Figure 13 shows A→d (♭4–) on the same circles. This move *does* change quality, and therefore $\Delta\rho$ becomes decoupled from $\Delta\mu$. The $\Delta\rho$ in this case is three steps closer than the $\Delta\mu$. The impact of this decoupling causes the ♭– progression to be *less* harmonically distant than the ♯4 progression, as this close root motion of ♭1 sonically rectifies the $\Delta\mu$ of ♭4. For all mode-reversing relationships, the root's distance will either be three steps further (+) or three steps closer (–) than the harmonic distance. The plus and minus signs serve a dual function in that they can represent both the raised or lowered $\Delta\varphi$ and $\Delta\rho$. The A→d as seen in Figure 13 possesses a $\Delta\mu$ of ♭4 and a $\Delta\rho$ of ♭1. One mode-preserving variation of this progression - A→D possesses both a $\Delta\mu$ and a $\Delta\rho$ of ♭1, and the other variation - A→F possesses both a $\Delta\mu$ and a $\Delta\rho$ of ♭4. Observing the interaction between $\Delta\mu$ and $\Delta\rho$ shows that A→d (♭4–) has commonalities with both A→D (♭1) *and* A→F (♭4). ♭4– and ♭1 possess identical $\Delta\rho$ distances, while ♭4– and ♭4 possess identical $\Delta\mu$ distances. This is the case for all progressions of major and minor triads.

When seeing a progression with a minus symbol - ♯5– as a new example - one can extrapolate that this progression, while harmonically it resembles a ♯5 motion, additionally has a major property in common with the mode-preserving progression that is three steps less: ♯2. The ♯5– therefore shares properties with both ♯2 and ♯5. The ♯2+ progression also shares properties with both ♯2 and ♯5, making a network of relations between these four similar progressions: ♯2, ♯2+, ♯5–, and ♯5. Figure 14 illustrates how these four progressions are linked by their common properties. Harmonic distance, circular mean distance, and root distance are important factors of the qualification of triadic progressions. The interaction between $\Delta\mu$, $\Delta\varphi$, and $\Delta\rho$ is a major property in how chord progressions move through harmonic space.

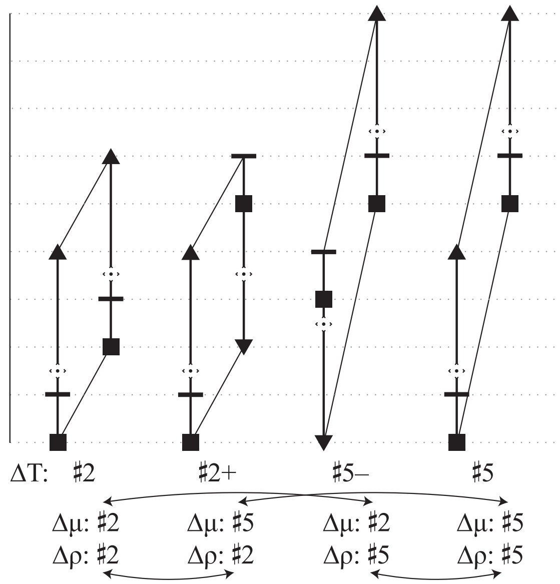


Figure 14: ♯2, ♯2+, ♯5–, and ♯5 on a generic Harmonic Proximity Graph, showing the similar properties between them. The distances of ♯2+ and ♯5– exist on a continuum between that of ♯2 and ♯5, where ♯2+ is more distant than ♯2 (hence the +), and ♯5 is less distant than ♯5.

In most cases, it becomes useful to track $\Delta\rho$ in addition to ΔT. Root motion $\Delta\rho$ can be qualified in a unique manner that relates similar intervals to one another. Root motion can be thought of as the following:

- **P** = Parallel. Zero semitones.
- **s** = Step of ± one semitone.
- **S** = Step of ± two semitones.
- **m** = Mediant of ± three semitones.
- **M** = Mediant of ± four semitones.
- **Q** = Quartal. ± five semitones.
- **T** = Tritone. Six semitones.

Figure 15 describes the how root distance can be classified as P for Parallel, S/s for Step, M/m for Mediant, Q for Quartal, and T for Tritone. For Step and Mediant root distances, the lowercase letter describes the smaller distance, while the uppercase letter describes the larger. Qualifying $\Delta\rho$ in this manner connects root motion to both voice leading and interval class. The respective range of P-T corresponds to the qualification of root motion by interval class from 0-6. The same information that is derived by looking at interval class can be seen in this lettered rendering of root motion. This nomenclature highlights the similar voice leading characteristics inherent in each root motion type. Figure 16 details how each root motion type affects voice leading and common tones. In the least diatonic version of each type, the horizontal bars will be common tones. In the most chromatic version of each type, the horizontal bars will be chromatically altered. By doing so, we begin to derive a way to refer to multiple non-identical progressions by their similar properties.

Δρ	T	Q	M	m	S	s	P	s	S	m	M	Q	T
Semitones	6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
Interval Class	6	5	4	3	2	1	0	1	2	3	4	5	6
Interval	T	P4	M3	m3	M2	m2	U	m2	M2	m3	M3	P4	T

Figure 15: Root distance described numerically, using the major/minor interval system, and with the proposed letter system.

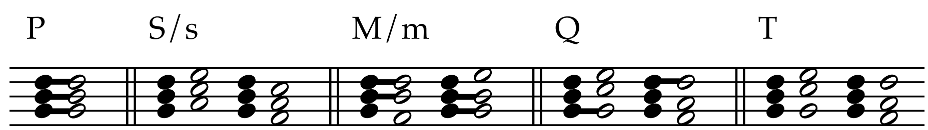


Figure 16: A general representation of the five triad relationship families and their unique voice leading characteristics: P = Parallel, S = Stepwise, M = Mediant, Q = Quartal, T = Tritone

Most of the following analyses of triadic music will use both $\Delta\rho$ and ΔT together in order to describe the interaction of three important parameters of harmonic motion: ρ, μ, and φ. As follows, $\Delta\rho$ will be expressed in its lettered form, with the value of ΔT (or $\Delta\varphi$ in non-triadic progressions) appended in superscript: $\Delta\rho^{\Delta T}$. As an example, The progression of *I'm Not the Only One* (from Figure 6) is: $\mathrm{M}^{\sharp 4} \rightarrow \mathrm{Q}^{\flat 4-} \rightarrow \mathrm{M}^{\flat 1+} \rightarrow \mathrm{Q}^{\sharp 1}$. This notation is seen later in Figure 33.

2.2 Harmonic Turbulence in *Creep*

In the final example of Figure 6, we notice the most harmonically turbulent variation of the I - iii - vi - IV in Radiohead's *Creep*. The progression contains the same opening $\mathrm{M}^{\sharp 4}$ as *I'm Not the Only One*, but instead of the negating $\mathrm{Q}^{\flat 4-}$, is followed instead by a move of $\mathrm{s}^{\flat 5}$. When using the transformation system, this transformation is *LPRP*, showing that the move in question is distant on both the *Tonnetz* and the Harmonic Proximity Graph. The mode-preserving quality of this motion also means that the harmonic distance and root distance are both ♭5, adding a higher degree of distance to the motion. The use of $s^{\flat 5}$ instead of the $Q^{\flat 4-}$ from *I'm Not the Only One* additionally produces more forward trajectory in the progression, as the $s^{\flat 5}$ moves to a chord that is not in the same space at the tonic, as the previous example had moved $Q^{\flat 4-}$ to the relative minor of the tonic.

2.2.1 Voice Leading and Harmonic Proximity

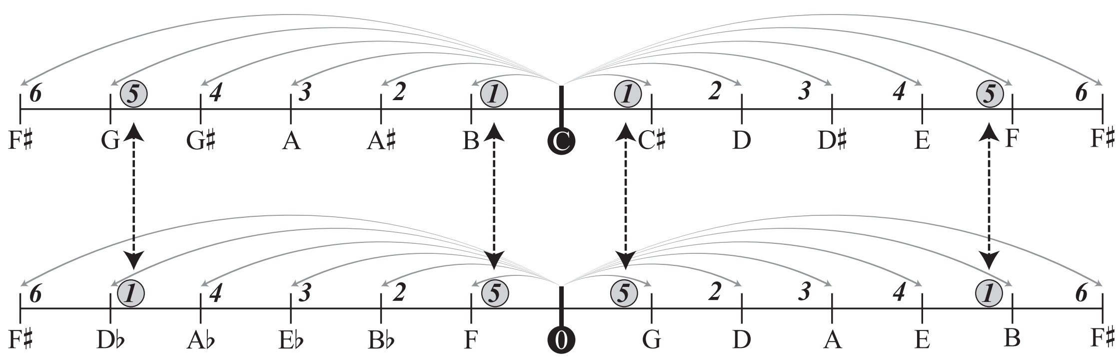


Figure 17: A comparison of minimum distances between a generation of semitones (top) and a generation of fifths (fifths). Most of the distances are the same, with the exception of 1 and 5 being swapped between the different generations.

Immediately following the $s^{\flat 5}$ is another flatward move of $P^{\flat 3-}$. This particular move is the harmonic result of a *P*, or Parallel transformation. *P* is seen as one of the most parsimonious transformations as it contains two common tones, and the third must only move by a semitone. The horizontal bars connecting the root and fifth between the C major and C minor triad highlight the voice leading quality of the Parallel transformation, while also showing exactly how far in this space the third must move. The primary reason why parsimonious voice leading and harmonic distance differ visually is due to how intervals - especially semitones - lie on the circle of fifths. On the top of Figure 17 is a horizontal space consisting of semitones. On the bottom is a similar space consisting of fifths. The arrows and bold numbers show the minimum distance in semitones from an arbitrary center point (in this example, the center point is C). What is intriguing about these spaces is that the semitonal distances between both spaces are *almost* identical. The only difference is that the position of the distance of 5 and 1 are flipped between the semitonal and the fifths spaces. A move of two steps, for example, would be equally as distant in both the spaces. This is the same for moves of three, four, and six. Moving one step on the semitone space is equal to moving five steps on the fifths space, and moving one step on the fifths space is equal to moving one step in the semitone space. The primary difference between voice leading proximity and harmonic proximity lies within this difference. The third of *Creep's* C major chord: E, must move a distance of 1 in semitonal space to reach E♭, therefore it is close in voice leading space, validating the parsimony inherent in the P transformation. Conversely, that same move from E to E♭ in fifths space is five spaces away, rendering it quite distant in harmonic space. The organization of our Harmonic Proximity Graph effectively highlights both of these interactions in a way that one can derive meaningful voice leading information while also noting that there is a discrepancy between how semitones are conceptualized within voice leading space and harmonic space.

2.2.2 Compounding Harmonic Distance

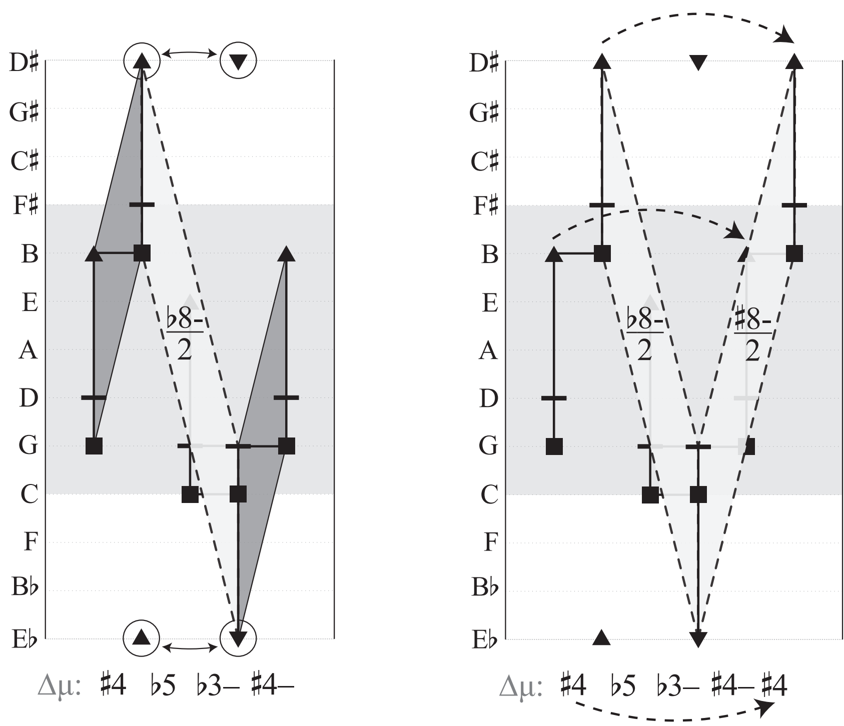


Figure 18: The progression from *Creep*, showing compounding harmonic distance of $\frac{\flat 8-}{2}$ and $\frac{\sharp 8-}{2}$.

The $s^{\flat 5}$ immediately being followed by a $P^{\flat 3-}$ creates a large harmonic trajectory which compounds flatward from B major to C minor. Without the intervening C major triad, a move from B→c would be sharpward by four steps, but the existence of the C major triad being placed between the two creates a flatward pull of $\frac{\flat 8-}{2}$ (Figure 18, Left). This continues the forward trajectory of the progression. From the opening chord hinting at a key of G major, moving immediately sharpward out of the diatonic span to only move equally flatward of the span creates a high sense of turbulence within the harmony, as there are hardly any points of stasis within the diatonic span of G major. The C major - being a IV chord in G - would be considered a point at where the span is validated, but with its approach from a chord five steps sharpward and its departure to a parallel three steps flatward, makes it difficult to consider this C major a true validation point. The right graph of Figure 18 shows how if the loop is extended, the combination of the last move from c→G and the first move from G→B (which are temporally adjacent because of the loop) create an equally distant compound motion of $\frac{\sharp 8-}{2}$.

2.2.3 Enharmonic Inequivalence

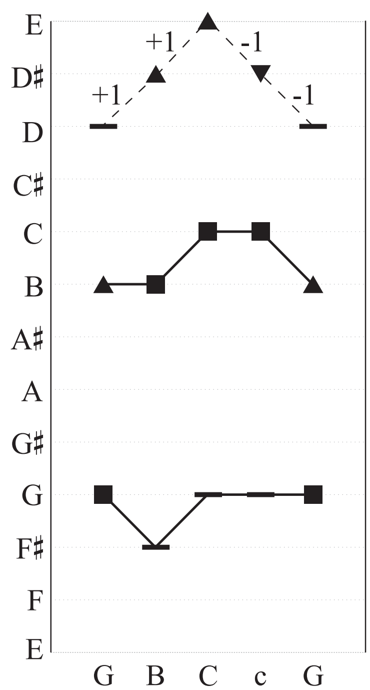


Figure 19: Voice Leading Proximity Graph of *Creep*, showing Doll's $\hat{5} \rightarrow \sharp\hat{5} \rightarrow \hat{6} \rightarrow \flat\hat{6} \rightarrow$ schema.

The progression in Radiohead's *Creep* includes an important voice leading schema of $\hat{5} \rightarrow \sharp\hat{5} \rightarrow \hat{6} \rightarrow \flat\hat{6} \rightarrow \hat{5}$ found in many rock songs. Christopher Doll explains the ubiquity of this particular motion in *Hearing Harmony*[3]. Figure 19 highlights the schema from D→D♯→E→E♭→D. The salience of this schema comes from the D♯ and the E♭. As enharmonics, these notes are identical

3. Christopher Doll, *Hearing Harmony: Toward a Tonal Theory for the Rock Era* (University of Michigan Press, 2017)

when using 12-Tone Equal Temperament. Figure 19 shows these notes as triangles in opposing directions as to highlight the quality of the full triad from B major (with a D♯) to C minor (with an E♭). There has been debate as to whether enharmonics should be seen as truly equal or not, and that their usage be purely for convenience, rather than to explain a deeper property of the music. Jason Yust argues that within the context of a center point (for example, a diatonic center), enharmonics are not a difference in position (as some argue that because of the 12TET system's approximation of just intonation, the D♯ in this example is technically slightly lower in pitch than the E♭), but a difference in orientation with respect to the center point[4].

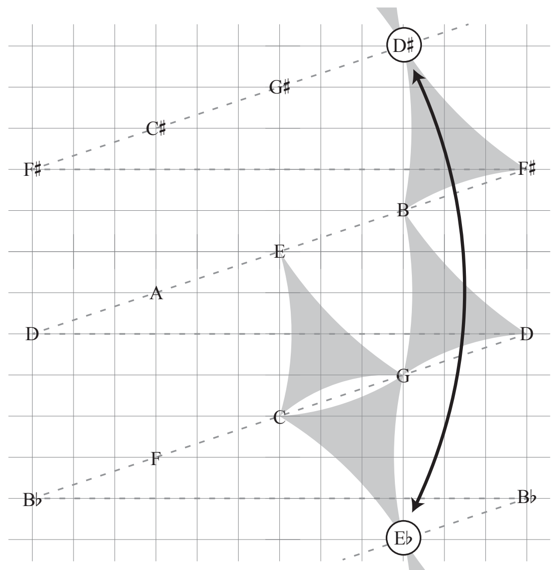


Figure 20: *Creep* mapped onto Yust's toroidal *Tonnetz*, showing the difference in positions of D♯ and E♭.

Figure 20 shows the *Creep* progression mapped onto Yust's toroidal *Tonnetz*. This particular *Tonnetz* is used to highlight movement of enharmonics in space. Any continuous motion upward or downward on the torus will result in a necessary enharmonic shift. The mapping of the *Creep* progression showcases the difference in position of D♯ and E♭ in regard to the tonic triad of G major as the center: The D♯ is sharpward, and the E♭ is flatward. On Figure 19, we see the D♯ and E♭ sharing the same horizontal position, and not a unique position as in the torus of Figure 20. Yust's torus is needed to accurately see that these pitches have different directions from the center point of G, and therefore have unique qualities between them. The diagonal axis of Yust's torus is a generation of T_7 upward, and T_5 downward, which aligns with the vertical axis of the Harmonic Proximity Graph. Because of this, the torus can be flattened along its opposing axis to derive

4. Jason Yust, *Organized Time: Rhythm, Tonality, and Form* (Oxford University Press, 2018), 254.

the Harmonic Proximity Graph. Figure 21 shows how the Harmonic Proximity Graph indeed can highlight the same properties of enharmonicism that are evident on the toroidal Tonnetz, with the D♯ on the top of the graph, and the E♭ on the bottom. Both the toroidal Tonnetz and the Harmonic Proximity Graph visually describe how the D♯ in *Creep* is heard as sharpward of the center, and the E♭ as flatward of the center.

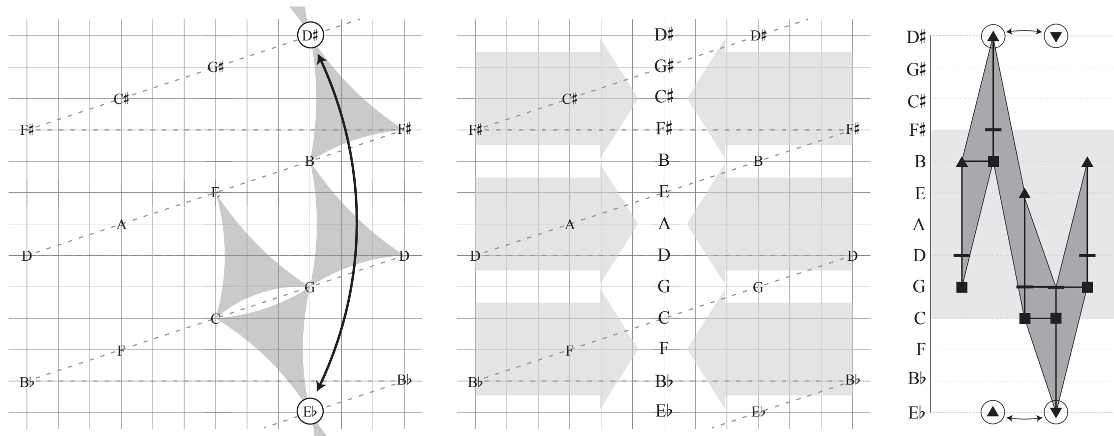


Figure 21: (Left) Figure 20. (Center) the collapsing of Yust's *Tonnetz* along the horizontal axis, revealing a vertical $\bigcirc P$. (Right) *Creep* mapped onto the vertical $\bigcirc P$ (*Harmonic Proximity*) graph, confirming the same difference between D♯ and E♭.

The duality of $\sharp\hat{5}$ and $\flat\hat{6}$ is a common chromatic figure in western musical language. It is typical to see progressions move chromatically sharpward via secondary dominants, and flatward via parallel borrowing. For example, the most common secondary dominant, V/V contains the first sharpward chromatic pitch on the Harmonic Proximity Graph: $\sharp\hat{4}$. Conversely, ♭VII and v both use the first flatward chromatic pitch, $\flat\hat{7}$. Moving equidistant by sharpward secondary dominants and flatward parallel borrowing, the first enharmonic one arrives at is $\sharp\hat{5}/\flat\hat{6}$. The $\sharp\hat{5}$ comes from the V/vi chord, and the $\flat\hat{6}$ is from either the ♭VI chord or the iv chord (as seen in *Creep*). Figure 22 illustrates this equidistant harmonic motion of ♯/♭4 that separates the $\sharp\hat{5}/\flat\hat{6}$ enharmonic into discrete scale degrees.

3 Negotiating Voice Leading and Harmonic Proximity

The layout of the *Tonnetz* attempts to negotiate between voice leading proximity and harmonic proximity. It is designed to that triangles sharing two corners (or one side) share two common tones, while triangles that meet at one corner share one common tone. Figure 23 is a *Tonnetz* that is broken into horizontal strips. When combining each triad and its respective R (relative)

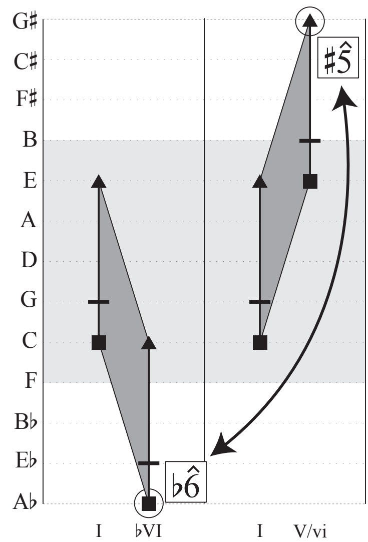


Figure 22: Typical flatward and sharpward motion by parallel minor borrowing and secondary dominants respectively. Moving equidistant in both directions by four steps reveals the first enharmonic: ♭6/♯5.

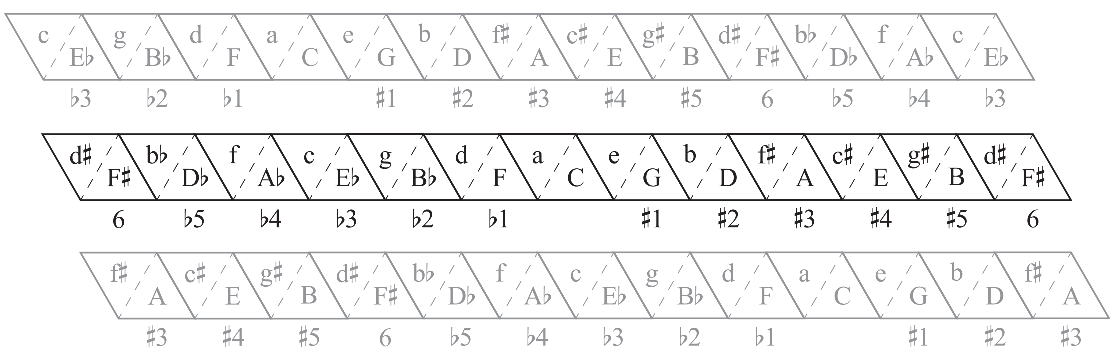


Figure 23: A dissected Tonnetz highlighting the circle of fifths, derived by using only R and L transformations. P and S shift the harmonic distance by ♭/♯3 (P) or ♯/♭4 (S).

transformation - effectively turning each triangle into a rhombus - the horizontal axis of the Tonnetz aligns with the $\bigcirc T$ triadic circle of fifths. This horizontal axis maps flatward motion to the left, and sharpward motion to the right. Using only R and L transformations correlates to harmonic distance around the circle of fifths, but not direction. Each compound RL transformation from a major chord corresponds to ♭1 harmonic motion, while from a minor chord the same RL moves ♯1. LR produces the opposite motion, where from major, LR moves ♯1, and from minor, LR moves ♭1.

Interestingly enough, in voice leading space these transformations also only dictate voice leading distance but not direction. *RL* moves three semitones up from major and three semitones down from minor. This is due to the inherent properties of the mathematical group in which the transformation system functions. Riemann's idea of dualism between major and minor triads dictates that minor triads are derived from an inverted overtone series, rendering them "upside-down" or inverted major triads[5]. This dualism was kept as a defining feature of Neo-Riemannian transformations developed primarily by Lewin[6] and Cohn[7].

3.0.1 Harmonic Proximity and Direction Through Transformations

A more concise transformation that does in fact communicate both harmonic distance and direction is David Lewin's *D*. This transformation[8] acts differently than *R* and *L* upon major and minor triads, where *D* moves the same direction on the Tonnetz whether the former triad is major or minor. As an example, a move from C major to B♭ major ($S^{\flat 2}$ according to my system), would be *RL* + *RL*, or D^2. The 2 in the superscript of this particular *D* transformation is actually synonymous with the circle of fifths flatward distance. Conversely, a move from B♭ to C (♯2) would be D^{-2}, where the negative superscript value shows circle of fifths sharpward distance. Because of *D*'s denial of the Riemann's major/minor duality, it does not function within the Neo-Riemannian group. There are other transformations that also deny the major/minor duality. Many of David Kopp's proposed transformations[9] function more similarly to Lewin's *D* than the Neo-Riemannian group. Much has been said about the logical complexities of intermingling transformations that possess Riemann's duality and transformations that do not, as the mathematical properties between them differ.

The assumption of simple transitivity is fairly ubiquitous in triadic transformational systems as a whole. With this property intact, there is only one transformation type per any two triads. All of the Neo-Riemannian transformations (*R, L, P, S, N, H*) create a simply transitive group. As soon as *D* is added to this group, there become multiple unique ways to transform one triad into another triad. When one combines dualistic and non-dualistic transformations, the larger mathematical group loses the property of simple transitivity. Herein lies the reason why *D* is not equivalent *RL* or *LR* although they may seem to be equal. John Rahn argues that the loss of simple transitivity within larger groups is not a negative effect[10]. One can combine Lewin's *D* and the extant *R* transformation to specifically highlight how mode-reversing transformations move in harmonic space. This is only

5. Hugo Riemann and Henry Bewerunge, *Harmony Simplified, or The Theory of the Tonal Functions of Chords* (Augener & Co., 1895)

6. David Lewin, *Generalized Musical Intervals and Transformations* (Oxford University Press, 2007)

7. Richard Cohn, "Weitzmann's Regions, My Cycles, and Douthett's Dancing Cubes," *Music Theory Spectrum* 22, no. 1 (1984): 89–103

8. Lewin, *Generalized Musical Intervals and Transformations*

9. David Kopp, *Chromatic Transformations in Nineteenth-Century Music* (Cambridge University Press, 2002)

10. John Rahn, "Cool Tools: Polysemic and Non-commutative Nets, Subchain Decompositions and Cross-projecting Pre-orders, Object-graphs, Chain-hom-sets and Chain-label-hom-sets, Forgetful Functors, Free Categories of a Net, and Ghosts," *Journal of Mathematics and Music* 1, no. 1 (2007): 1–32

possible because R does not possess any corresponding harmonic distance, and therefore does not interfere with the logical difference between how Lewin's D affects major or minor triads. C major to G minor (♭2–) would be D^2R, showing that the progression "moved from a major triad to a minor triad two steps flatward", while G minor to C major (Q$^{♯2–}$) would be $D^{-2}R$. What is missing from this particular method in relation to mine is that my method uses positive and negative symbols to show exactly how the root motion of mode-reversing progressions *affects* the property of harmonic motion, while the D + R system simply explains whether a transformation has changed mode or not.

TRANSFORMATION	HARMONIC MOVEMENT	TRANSFORMATION	HARMONIC MOVEMENT
R	0	P	♯/♭3
L	♯/♭1	S	♯/♭4

TRANSFORMATION	HARMONIC MOVEMENT
RL	♯/♭1
LR	♯/♭1
D	♭1
D^{-1}	♯1

TRANSFORMATION (D)	SIMPLEST (RLPSN)	HARMONIC MOVEMENT
D^4	PL	♭4
D^{-4}	SR	♯4

Table 2: 1. Simple Neo-Riemannian transformations and their harmonic distances. 2. RL, LR, and D transformations and their harmonic motions. 3. Transformations using Lewin's D and their harmonic motions.

3.0.2 *P* and *S*'s Affect on Harmonic Distance

The upper and lower horizontal strips in Figure 23 show the harmonic wormhole effect that happens when applying vertical P or S transformations. Applying P to C major (C→c) moves the harmonic center by 3 fifth-steps, and applying S (C→c♯) moves the harmonic center by 4 fifth-steps. Table

2 shows this connection between the *L, R, P, & S* transformations and their respective harmonic distances. Figure 24 shows the same four transformations on a generic version of the Harmonic Proximity Graph. The same harmonic distances outlined in both Figure 23 and Table 2 are found on this graph, especially emphasizing the relative chromaticism found in the *P* and *S* transformation. Although the *Tonnetz* is not typically used in such a manner, knowing the harmonic distance inherent in these transformations helps to have some idea of harmonic trajectory when mapping progressions onto the *Tonnetz* or when analyzing a progression using the transformation system. As previously discussed, in many progressions, there are differences in results between voice leading proximity and harmonic proximity. This is due in part to the change in the relative proximity of one semitone and five semitones when observing semitone and fifths spaces (detailed in Figure 17).

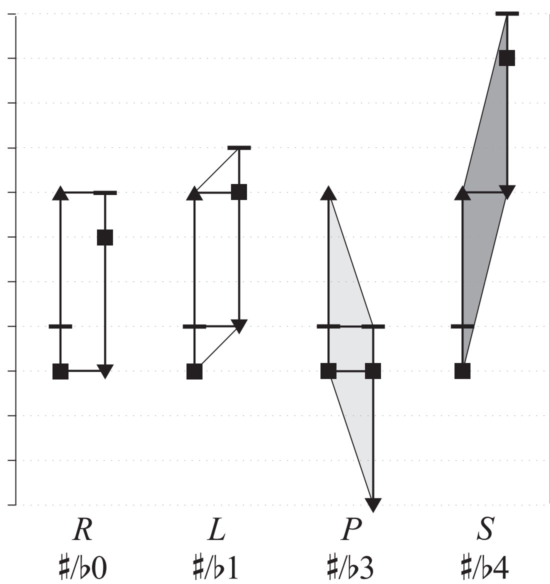


Figure 24: *R, L, P, & S* transformations on a generic Harmonic Proximity Graph.

3.1 The *Cube Dance*

In regard to the efficacy of quantifying voice leading proximity, Dmitri Tymoczko argues that the *Tonnetz* is ineffective its ability to measure such proximity, and proves that *Tonnetz* distance is not equal to voice leading distance[11]. He then argues that Douthett and Steinbach's *Cube Dance*[12] is a more effective tool in measuring voice leading distance. Although by doing so, the Cube Dance

11. Dmitri Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (Oxford University Press, 2011), 412.
12. Jack Douthett and Peter Steinbach, "Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition," *Journal of Music Theory* 42, no. 2 (1998): 241–263, 254.

divorces itself from this negotiation between voice leading and harmonic proximity, and instead leans into the exact reasons for such differences between spaces outlined in earlier Figure 17. With the exception of R and L, progressions that possess strict, parsimonious voice leading additionally possess a more distant harmonic relationship.

3.1.1 Cube Dance Progressions in Harmonic Space

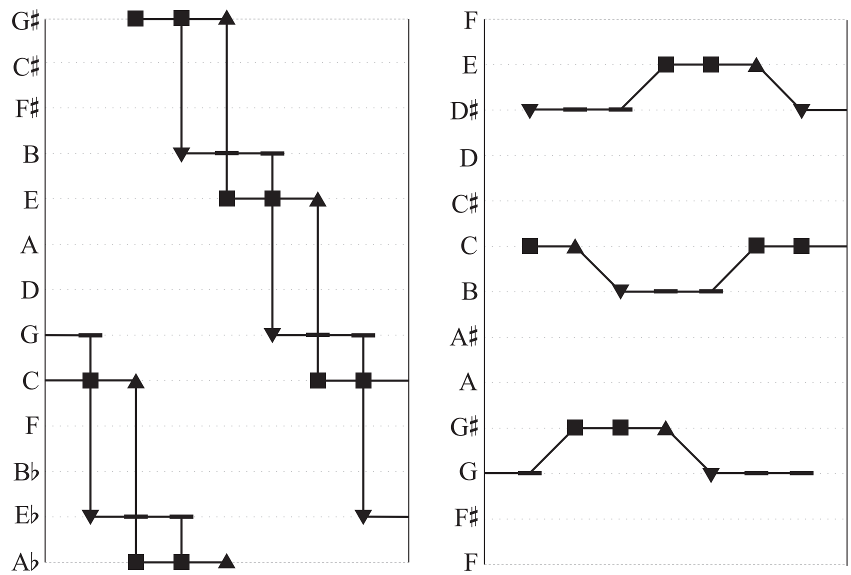


Figure 25: (Left) The progression from the six *Klänge* in the Northern Cycle (inspiration for the Cube Dance) mapped onto the Harmonic Proximity Graph, showing the high level of harmonic distance. (Right) The same progression mapped onto a semitone variant of the graph, showing the high level of parsimony.

The progression in Figure 25 is from Richard Cohn's article, demonstrating all the triads in one area of the Cube Dance are connected via voice leading by only one semitone[13]. The left graph visualizes how the progression moves violently in harmonic space, even completing one entire harmonic cycle. While the adjacent relationships move by either $M^{\flat 1+}$ or $P^{\flat 3-}$, all of the triads in this progression played in any order would be considered highly parsimonious, as they are all in the same region on the Cube Dance. Figure 26 shows the difference in harmonic spans between one of the regions of the Cube Dance versus the triads of the C major diatonic set. The Cube Dance is constructed so that each cube is connected creating a Weitzmann region. These regions are special because each of the six triads in each Weitzmann region all possess the same voice leading proximity of two semitones. Figure 27 again compares the Weitzmann region (containing

13. Cohn, "Weitzmann's Regions, My Cycles, and Douthett's Dancing Cubes," 96.

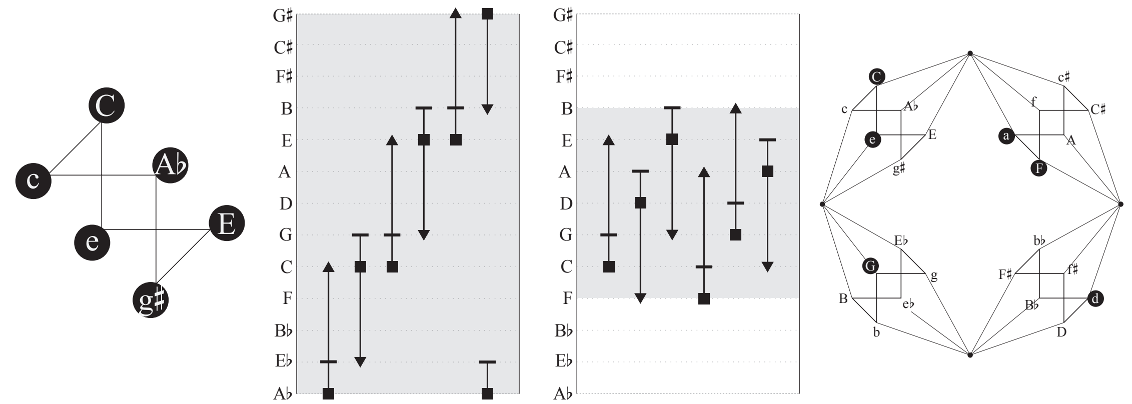


Figure 26: (From left to right) A *PL* cycle, visualizing on the cube dance. The same *PL* cycle on the Harmonic Proximity Graph. The Major and Minor chords in a C major scale on the Harmonic Proximity Graph. The same chords on the Cube Dance.

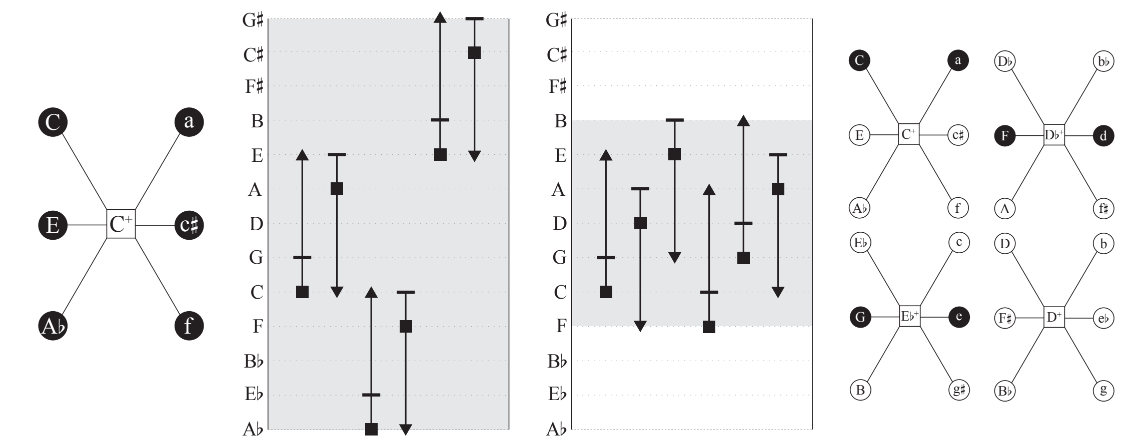


Figure 27: (From left to right) The C$^+$ Weitzmann region. The same Weitzmann region mapped on the Harmonic Proximity Graph. The Major and Minor chords in a C major scale on the Harmonic Proximity Graph. The same chords shown on the four Weitzmann regions.

C, a, A♭, f, E, and c♯) and its harmonic span to the diatonic set. Between these three figures, the volatility of the left graph compared to the extremely concise motion of the right graph directly shows the aforementioned discrepancy between harmonic proximity and voice leading proximity. It is also clear that voice leading proximity and harmonic proximity are not only different, but typically oppose one another. The Harmonic Proximity Graph shows how progressions that have concise voice leading tend to possess unique harmonic properties, and why such semitonal voice leading impacts the harmonic quality of these progressions. The Harmonic Proximity Graph, and when necessary, the *Voice Leading Proximity Graph* (seen in Figure 7 (right), Figure 19, and Figure

25) can be combined to show a full representation of the properties inherent in chord progressions.

3.2 Voice Leading on the Harmonic Proximity Graph

Figure 28: The vertical axis of the *Harmonic Proximity Graph*, showing the minimum voice leading distance between one note and all other notes. This ordering of voice leading distances (5, 2, 3, 4, 1, 6) aligns with Figure 17 and shows the similarities and differences in proximity between when comparing fifths space to semitone space.

While the Cube Dance and the Voice Leading Proximity Graph directly show how triads move in semitonal space, the Harmonic Proximity Graph can also be used to derive the same information while maintaining all of the salient information about harmonic proximity. Figure 28 shows the vertical axis of the Harmonic Proximity Graph with arrows that show the minimum distance by semitones from one note to all other notes on the graph. The order of these distances matches those of the fifths-generated graph in Figure 17. With the exception of the flipped positions of 5 and 2, all of the other distances are in numerical order, and therefore correspond to the amount of spaces on the vertical axis of the Harmonic Proximity Graph: 1 space = 5, 2 = 2, 3 = 3, 4 = 4, 1 = 5, and 6 = 6.

When measuring voice leading on the Harmonic Proximity Graph, the smallest distance between one note and another note correlates to the smallest of all the possible semitonal distances from one note of the former triad to all of the notes of the latter triad. Assuming that only efficient voice leading is being analyzed, the furthest any voice can move is by three semitones, rendering the distances of four or greater on Figure 28 is trivial. Figure 29 (left) shows the progression C→d on the Harmonic Proximity Graph. The minimum voice leading distances between the root of the former triad - C - to the three notes of the latter triad are C→D (2 semitones), C→A (3), C→F (5), with the smallest distance being 2. From the fifth of the former - G - to the latter triad is G→D (5), G→A (2), G→F (2); the smallest distance here also being 2. The third of the former triad - E - to the F in the latter triad is 1 semitone, which is smallest by default. Adding the smallest distances from each note of the latter triad will always give the same amount of voice leading distance as the Cube Dance and the Voice Leading Proximity Graph (also in Figure 29). When visualizing this property of voice leading on the Harmonic Proximity Graph, it becomes quite easy to count

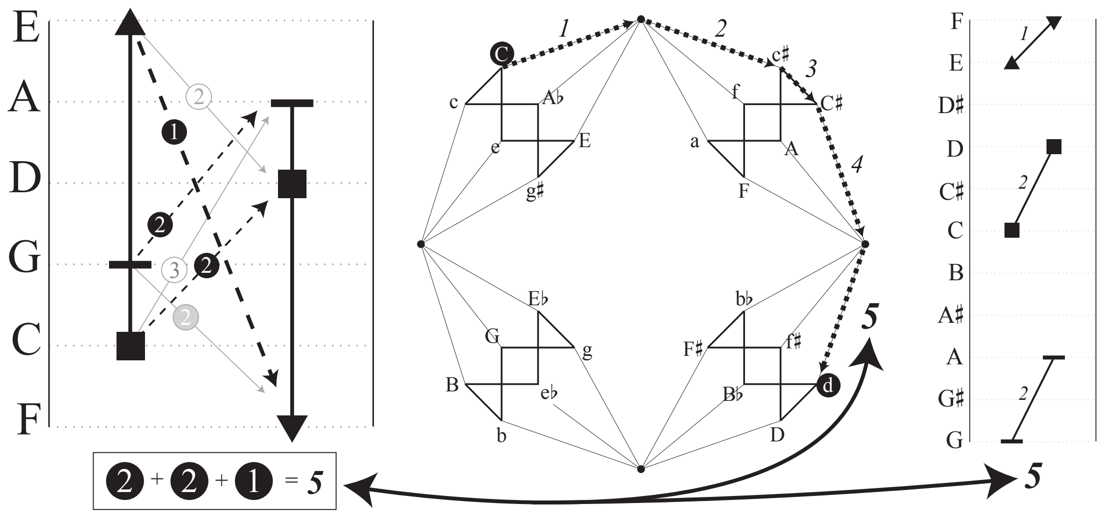


Figure 29: (Left) A method for finding the voice leading proximity from any two adjacent triads (eg. C→d) by summing the smallest distance from each note of the former to each note of the latter triad. (Center) C major to D minor on the Cube Dance, confirming the 5 necessary voice leading steps between the two triads. (Right) C major to D minor mapped onto my voice leading graph, also confirming the 5 necessary voice leading steps.

the distances with the prerequisite that only distances of 1, 2, and 3 are of interest, and that five spaces on the Harmonic Proximity Graph is equal to one semitone of distance. With the "5=1" conversion being accounted for, simply counting the spaces between the notes will yield the correct distance of two or three.

3.3 *The Gunner's Dream*

Another example of a modified I-iii-vi-IV progression from earlier exists in Pink Floyd's *The Gunner's Dream* (1983). In this variation, the I moves to an augmented triad consisting of $\hat{1}$, $\hat{3}$, and $\sharp\hat{5}$ (I^{+} in this case). The entire progression maps very neatly onto the Cube Dance (seen in Figure 30), as the defining property of this model is that each cube is connected via an augmented triad. This progression - I→I^{+}→vi→IV - is extremely parsimonious, as each move between chords only needs one semitone of motion to complete. Figure 30 shows how one can derive the same information about voice leading proximity while also viewing all of the unique harmonic properties on the Harmonic Proximity Graph.

The inclusion of the augmented triad in this progression makes it more difficult to measure a harmonic distance to and from the chord. The augmented triad spans the entirety of the circle of fifths, and is perfectly balanced on the circle. There are two other common chords that share there

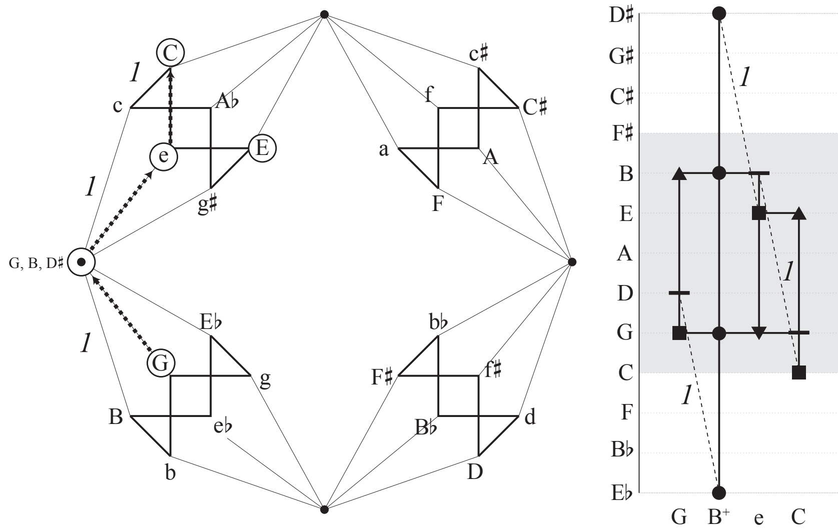


Figure 30: The opening progression from Pink Floyd's *The Gunner's Dream:* G→B$^+$→e→C.

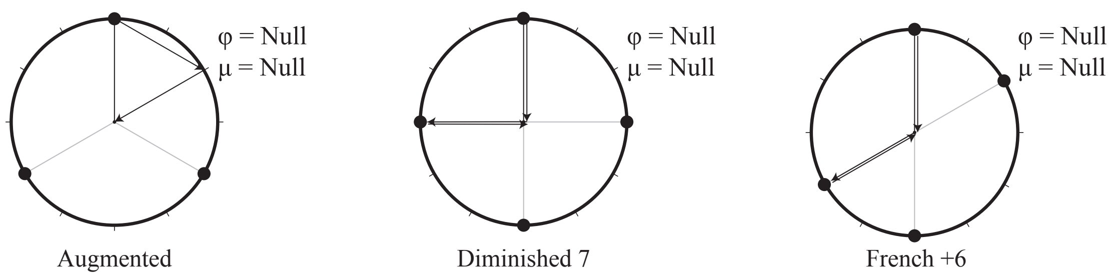
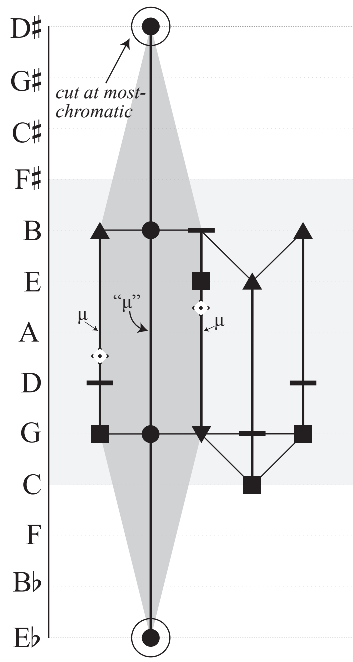


Figure 31: An augmented triad, a fully-diminished seventh chord, and a French Augmented Sixth chord mapped onto the circle of fifths. The chords are completely balanced, and therefore do not possess a value for φ nor μ. These chords have an ambiguous position on the circle of fifths.

properties: the fully-diminished seventh chord and the French augmented sixth chord. Figure 31 shows these three chords mapped onto the circle of fifths, showing how φ and μ are technically undefined for each of these chords.

When chords of this type arise, we can utilize the unique traits of μ and φ to "cut" the span at a certain point based on the contextual placement of the chord within the surrounding progression. Figure 32 shows how a μ value can be assigned to the augmented chord in *The Gunner's Dream* based on the context of the I and vi chords. The span of the augmented triad can be "cut" at the note which is the furthest point away from the μ and φ values of the neighboring chords. In this case, that is D♯/E♭. Creating this cut gives us a sharp-most and flat-most member of the chord (D♯/E♭ in both cases), which in turn gives us a μ value. By creating this contextual μ, we can then



	I		I⁺		vi		IV		I
Δμ		0		0		♭1		♯1	
Δφ			$\frac{\sharp 1}{2}$			♭2		♯1	
σ	4		12		4		4		4
L^2		√1		√1		√1		√5	

Figure 32: The progression from *The Gunner's Dream*, showing how a contextual cut can be made in order to derive a μ value for the completely balanced augmented triad.

describe the $\Delta\mu$ of the progression as 0→0. In these instances, the use of $\Delta\mu$ versus $\Delta\varphi$ becomes more unique. The φ circular mean measurement treats the circle of fifths as a true circular space, whereas the measurement of μ treats the circle of fifths as a unidimensional, linear space. Because the φ value is derived from a direct calculation of the circular mean of the notes on the circle of fifths, it is a true measurement of the weighted-center point of the chord in circular space. It is difficult to make a case for a *contextual* φ value for these types of chords. The μ measurement - being based on a linear representation of the circle of fifths - becomes helpful in understanding how chords can be interpreted contextually, effectively altering their μ values based on where their spans are cut according to harmonic context. As for the φ value, it is possible, and sometimes relevant, to measure the $\Delta\varphi$ of from the previous chord, through the ambiguous chord, to the following chord and express it as $\frac{\Delta\varphi}{2}$. These particular progressions showcase again the importance of decoupling $\Delta\mu$ and $\Delta\varphi$, and how their unique measurements show different aspects of the harmonic space which the progressions occupy.

Although the resultant $\Delta\mu$ of I$\rightarrow$I$^{+}\rightarrow$vi is 0 $\rightarrow$0, what is missing from this analysis is the fact that the augmented triad *is* more chromatic than the I and vi chords as it contains the note that is the furthest away from the diatonic span. Up until now, we have only qualified chromaticism through progressions that are temporally adjacent - which we can call *horizontal* chromaticism. The augmented chord - while it is possesses the same μ as the I and the vi - is *vertically* chromatic due to its large span of 12 in comparison to the other chords' spans of 4. This becomes an important distinction when talking about more complex chords, such as chords with extensions found ubiquitously in Jazz music.

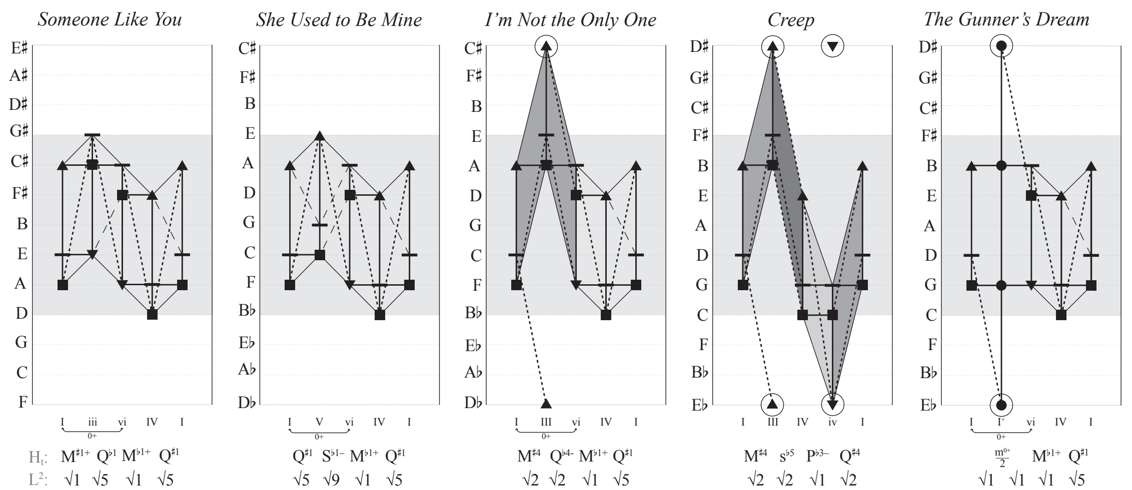


Figure 33: Five variations on the I-iii-vi-IV, with an additional layer on the Harmonic Proximity Graph showing voice leading properties. Thicker dashed lines with shorter dashes show semitonal voice leading, and thinner dashed lines with longer dashes show voice leading by more than a semitone. The voice leading amount can be derived by counting the number of vertical spaces between the notes (with the exception of 1 space being equal to 5 voice leading steps).

Figure 33 shows all of the variations of the aforementioned core progression, with an additional analytical layer that describes the voice leading characteristics of each progression. The slightly thicker, densely dashed line shows semitonal voice leading, while the thinner dashed line with longer dashes shows voice leading distances higher than a semitone. One is able to easily derive the voice leading distance of the long-dashed lines by counting the amount of spaces between the two notes. The long-dashed line will only show voice leading distances of two or three as maximally efficient voice leading is assumed.

The networks below each of the graphs in Figure 34 (and in Figure 34) are a condensed way to show the salient features of these progressions in both harmonic (H_t) and voice leading (L^2) spaces. As previously mentioned, the progressions from *Creep* and *I'm Not the Only One* possess higher

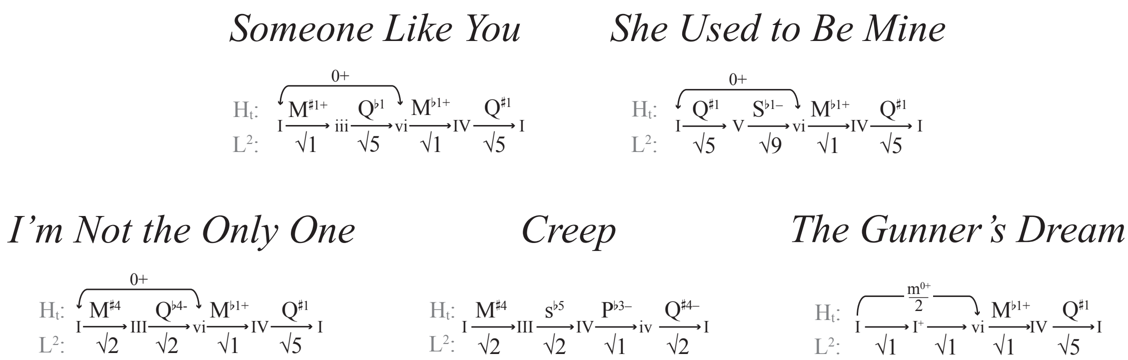


Figure 34: The five variations shown as networks describing both the harmonic distance and voice leading distance between adjacent triads.

levels of harmonic motion than *Someone Like You* and *She Used to Be Mine*. The total amount voice leading between *Someone Like You* and *I'm Not the Only One* is equal (8) using the L^1 vector norm. This is due to the fact that while I→iii takes one semitone as opposed to I→III's two, the consequent motion from iii→vi takes three semitones instead of III→vi's two. Using the L^2 vector norm shows a that *I'm Not the Only One*'s progression possess a slightly smaller total distance than *Someone Like You*, but the alteration of the iii chord in this instance has no major effect on the total amount of voice leading distance while having a major effect on the harmonic distance between these two progressions. *Creep's* progression possesses a smaller voice leading distance than any of the previous progressions, even though it possesses a much higher harmonic distance than the other three. *Creep* also only possesses purely semitonal voice leading motion. This property is almost true in *The Gunner's Dream*, but the move from IV→I requires a whole-step voice leading distance. Theorist Moreno Andreatta actually created a completely parsimonious version of *The Gunner's Dream* by altering the progression so that it completes a Hamiltonian path around the Cube Dance. This variation "fixes" the voice leading seam found in the original progression.

The *She Used to Be Mine* variation has the highest amount of voice leading steps, primarily due to the move from V→vi which has the largest voice leading distance of $\sqrt{9}$. This particular motion also possesses the *least* harmonic motion of $S^{\sharp 1-}$, further emphasizing the polarity between harmonic and voice leading proximities. The use of the augmented triad in *The Gunner's Dream* creates a highly concise voice leading movement that is not shared by any other progression. This progression also shares the same compound $\frac{0+}{2}$ harmonic motion from the first to the third chord as *Someone Like You*, *She Used to Be Mine*, and *I'm Not the Only One*. The claim that voice leading and harmonic distances possess a roughly inverse relation holds true through the progression of *The Gunner's Dream*. While the augmented chord does *not* create any horizontal chromaticism, it - by itself - is vertically chromatic. The augmented chord being used as a voice leading "bridge" between the I and vi creates a concise voice leading path, but sacrifices vertical chromaticism to do so.

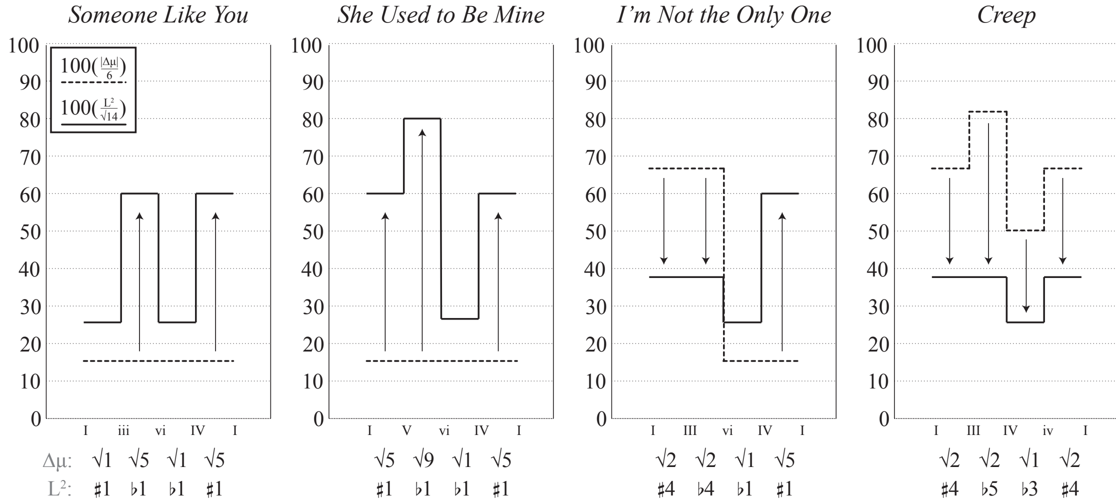


Figure 35: A comparison of normalized voice leading and harmonic distances.

Figure 35 directly compares the magnitudes of harmonic distance on the circle of fifths (dashed lines) against the L^2 voice leading distances (solid lines). The harmonic distance is normalized by taking the distance (without regard to the direction) out of 6 - the furthest possible distance, and multiplying it by 100. The voice leading distance is measured out of the furthest possible distance of $\sqrt{14}$, and then multiplied by 100 as well. What is created here is an easy way to see where low levels of voice leading distance result in high harmonic motion, and vice versa. The arrows on the graph show the points where the voice leading distance disagrees with the harmonic distance the most. The *Creep* progression possesses the most consistent opposing relationship between voice leading proximity and harmonic proximity, where in comparison to the other progressions, the voice leading line is the lowest, and the harmonic distance line is the highest. The progressions of *I'm Not the Only One* shows an interesting intersection, where when the harmonic distance is high ($M^{\sharp 4}$, $Q^{\flat 4-}$), the voice leading is low comparatively ($\sqrt{2}$, $\sqrt{2}$). As soon as the progression moves into the diatonic portion - $M^{\flat 1+}m\ Q^{\sharp 1}$ - we see the harmonic distance drop, and the voice leading distance rise above the harmonic distance ($\sqrt{1}$, $\sqrt{5}$). The progression of *She Used to Be Mine* possesses the furthest discrepancy between respective distances in the progression from V to vi. The voice leading is extremely distant, while the harmonic distance remains low.

Interestingly, the points on this graph where voice leading and harmonic distances seem to agree align with the most simple Neo-Riemannian transformations. For instance, in the first three progressions, the point at which the two distances come close to one another is the *L* transformation. The second half of the *Creep* progression also has closer distances (although they are still quite far), which are the Neo-Riemannian *P* and *N*. This could point to the fact that the *Tonnetz* as well as the Neo-Riemannian transformations in particular serve to negotiate between these two distances.

The interplay between voice leading proximity and harmonic proximity creates a balance that describes the most salient characteristics of triadic progressions. These two proximities are roughly inversely related, where progressions that exhibit close voice leading proximity often span considerable harmonic distances, and vice versa. Tymoczko references this phenomenon when discussing a highly chromatic passing chord in Schumann's *Chopin*, stating that because the motion is parsimonious, the resultant harmonic distance is quite far[14]. The *Tonnetz* is structured so that it combines the voice leading properties of the transformation system with harmonic motion through the circle of fifths, though the original transformations only give information about distance and not direction. Lewin's *D* comes closer to a transformation that navigates harmonic space, but in turn becomes more complicated to use alongside the original transformations as they do not function the same on major and minor triads. As previously observed from Figure 3, and further clarified by Tymoczko[15], there is a disconnect between voice leading distance on the Cube Dance and the *Tonnetz* that renders it (as well as the transformation system) relatively complicated in regard to its usability for analyzing either voice leading or harmonic proximities. While the Cube Dance effectively maps voice leading through semitonal space, The Harmonic Proximity Graph directly shows essential information about harmonic motion while maintaining the ability to derive the same voice leading information as the Cube Dance.

4 Relating Multiple Triadic Progressions

The *Proximity Network* in Figure 36 shows every possible progression between two major or minor triads, organized into a network that shows how each progression relates to another. There are three types of relations between progressions. The most obvious is the Retrograde (R) relationship. For any given progression, the retrograde will possess the same $\Delta\rho$, as well the same distance for ΔT, but will possess the opposite direction on the graph ($\flat \leftrightarrow \sharp$). The Proximity Network shows R relationships with solid, vertical lines. The second most salient relationship between progressions is the maintaining of $\Delta\mu$. For progressions in which $1 \geq \Delta\mu \leq 5$, there are exactly three progressions that possess each possible $\Delta\mu$ value. For progressions where $\Delta\mu = 1 \cup 6$, there are two progressions for each $\Delta\mu$ value. Because of the identical spans ($\sigma = 4$) of major and minor triads, as well as the fact that relative triads possess identical μ positions on the graph, possessing identical $\Delta\mu$ values is an important commonality between different progressions. For instance, $\mathrm{M}^{\sharp 4}$ (C→E) and $\mathrm{s}^{\sharp 4+}$

14. Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice*, 215-216.
15. Tymoczko, 412

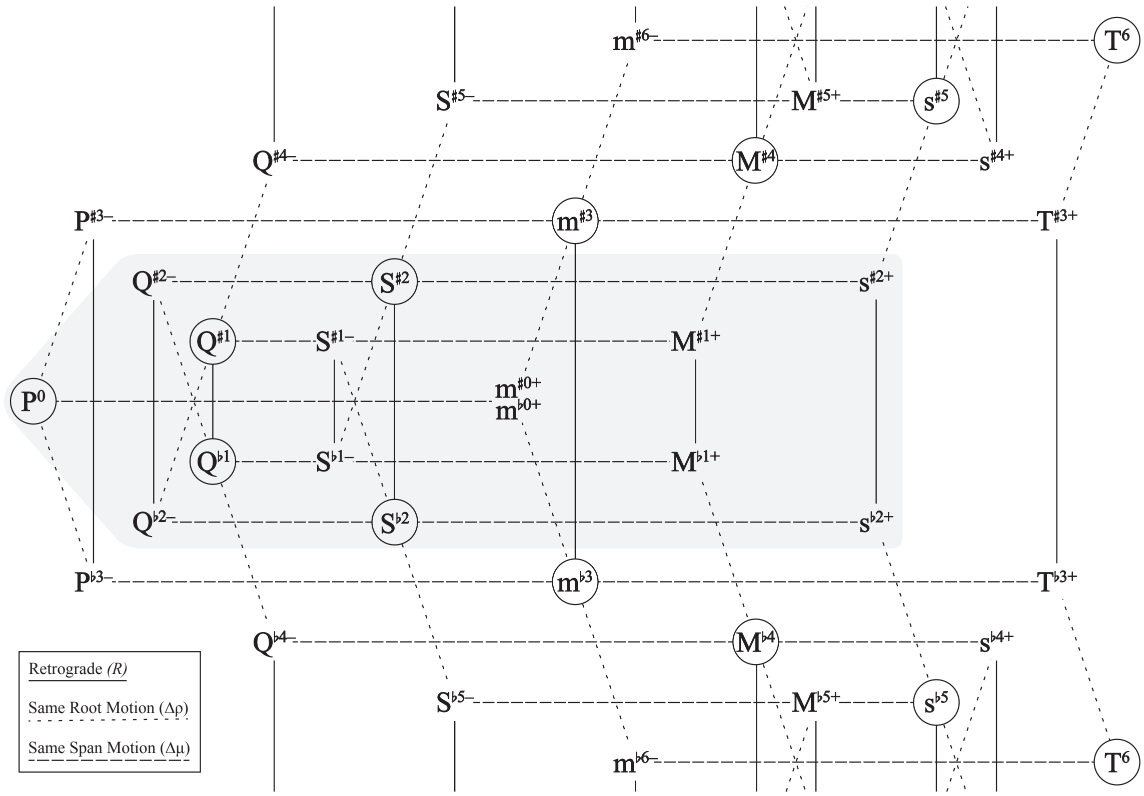


Figure 36: The *Triadic Progression Proximity Network*, showing all possible progressions between two chords, and their relationships to one another.

(C→c♯) are related by $\Delta\mu$: ♯4. This is because E major and C♯ minor have identical μ positions, being that they are relative triads. Either of these two progressions move the harmonic center in the same manner: ♯4. Progressions that are related by $\Delta\mu$ are shown on Figure 36 as horizontal dashed lines. The $\Delta\mu$ relation is also the only relation type that compares progressions with different $\Delta\rho$ values.

The final way in which triads can be related is by identical root motion, or $\Delta\rho$. In order to be related by $\Delta\rho$, the root motion needs to be in the same direction. For example, while C - E and C - A♭ are both "M" mediant progressions, they are in the same $\Delta\rho$ family but are more importantly related by R, which is a higher level of relation. On the other hand, C - E (M$^{\sharp 4}$), C - e (M$^{\sharp 1+}$), and c - E (M$^{\flat 5+}$) (and any transpositions of such progressions) are all related by $\Delta\rho$. The $\Delta\rho$ relations are shown on Figure 36 as dotted diagonal lines. For progressions in which $1 \geq \Delta\mu \leq 5$, each mode-preserving progression has one $\Delta\rho$ related progression that is three $\Delta\mu$ steps lower, and one that is three $\Delta\mu$ steps higher. Because the root motion is identical, the only property than changes between these three progressions is the quality of one of the chords; either

Maj→Maj (or Min→Min), Maj→Min, or Min→Maj. Changing the quality of a chord in a given progression either raises or lowers the $\Delta\mu$ harmonic distance by three steps because of the inherent $3^{\flat P-}/3^{\sharp P-}$ within the progression. The $\Delta\rho$ relation connects each set of three progressions by this particular property. In the previous analyses, the difference between *Someone Like You* and *I'm Not the Only One* was the I - iii versus the I - III. These two progressions are related by $\Delta\rho$: $M^{\sharp 1+}$ $\leftarrow\Delta\rho\rightarrow$ $M^{\sharp 4}$. Progressions in which $\Delta\mu = 1 \cup 6$ have a unique interaction with their $\Delta\rho$ relations. The P^0 progression is $\Delta\rho$ related to both $P^{\sharp 3-}$ and $P^{\flat 3-}$ because all three possess a $\Delta\rho$ of 0. This is the same for T^6, as its root motion is directionless: 6. T^6 is $\Delta\rho$ related to both $T^{\sharp 3+}$ and $T^{\flat 3+}$.

The four progressions from *Someone Like You*, *She Used to Be Mine*, *I'm Not the Only One*, and *Creep* are mapped onto the Proximity Network in Figure 37. This mapping gives a visualization of exactly how the progressions are similar. There is a high concentration of progressions towards the center of the network. This is because *Someone Like You* (solid line) and *She Used to Be Mine* (long-dashed line) are fully diatonic, meaning that they both only use progressions that possess a ΔT of 2 steps of less, as well as have progressions that negate any chromatic drift. Both of these diatonic progressions exist within the shaded gray diatonic area in the center. The progression from *Someone Like You* begins with $M^{\sharp 1+}$ moving to $Q^{\flat 1}$. The Network shows how the following two progressions: $M^{\flat 1+}$ and $Q^{\sharp 1}$ are related to the first two by R retrograde. *She Used to Be Mine* begins on a $Q^{\sharp 1}$ which is related to the $M^{\sharp 1+}$ by $\Delta\mu$. The progression then moves to $S^{\flat 1-}$, which is related to $Q^{\sharp 1}$ also by $\Delta\mu$. The two progressions then merge on the $M^{\flat 1+}$.

I'm Not the Only One begins on the $M^{\sharp 4}$ and moves to the $Q^{\flat 4-}$. These two progressions are much further from the center of the Proximity Network, as they are chromatic. The $M^{\sharp 4}$ of *I'm Not the Only One* is connected to the $M^{\sharp 1+}$ of *Someone Like You* and *She Used to Be Mine* by $\Delta\rho$ because both of these variations possess identical root motion (♯4). This is true for the following $Q^{\flat 4-}$ and $Q^{\flat 1}$, which are also related by $\Delta\rho$. The progression then also merges with the previous two on $M^{\flat 1+}$, joining the diatonic area of the network. Looking at the corresponding line for *I'm Not the Only One* shows how the progression begins chromatic - as its first two progressions are far from the center of the network - and then becomes diatonic during the second half of the loop. This progression moves through both diatonic and chromatic space.

Creep, on the other hand, moves solely through chromatic space. The progression forms a border around all of the other progressions on the network, showing a highly chromatic variation of the diatonic standards. This progression - like *I'm Not the Only One* - begins on $M^{\sharp 4}$, but moves instead to a $s^{\flat 5}$, which is not directly related to any of the other progressions. This disconnect continues when the progression moves to a $P^{\flat 3-}$. *Creep* then comes back to its connection with the other progressions with the final $Q^{\sharp 4-}$, connected to the final $Q^{\sharp 1}$ of the other progressions by $\Delta\rho$. *Creep* is definitely the most unique of the four progressions, but it still has subtle connections to the others that become easily visible when mapped on the Proximity Network. Figure 38 shows a concise network that similarly describes the connections between each progression as found on

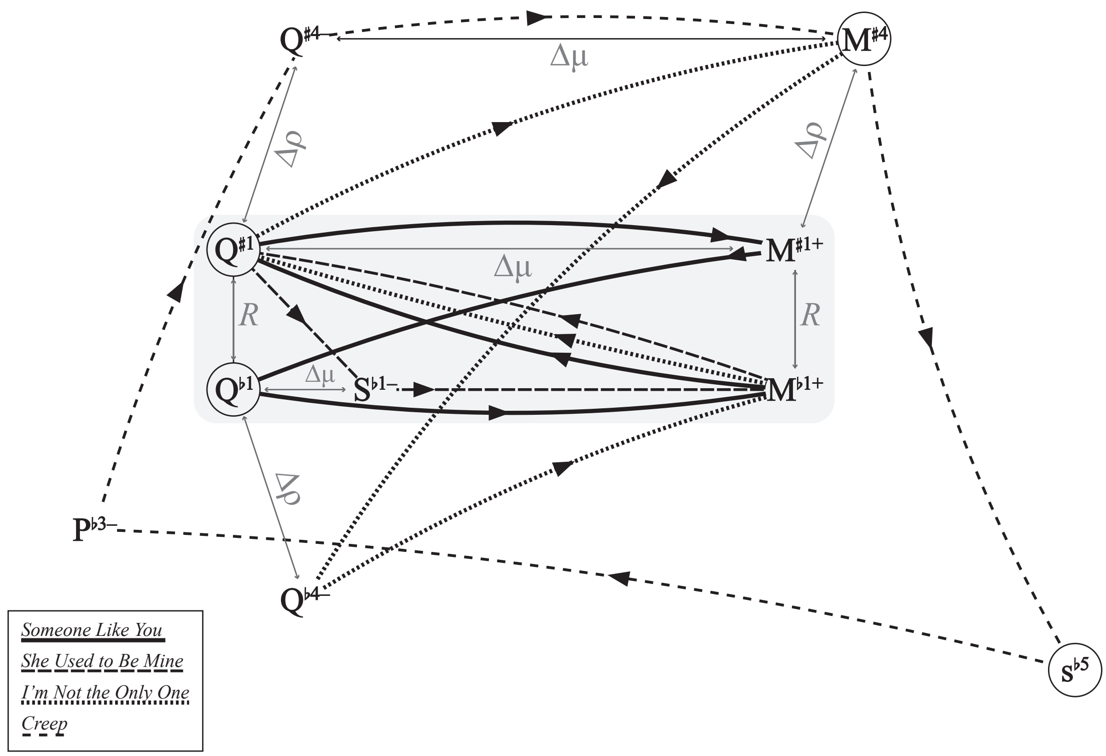


Figure 37: The four chord progressions from our previous comparative analysis mapped onto the Proximity Network.

the Proximity Network. All four of the progressions have a directional similarity in their harmonic motion of $\sharp \rightarrow \flat \rightarrow \flat \rightarrow \sharp$. This can also be seen on the Proximity Network, where from each starting point on the sharp side of the network (the top half), every progression then moves downward across the center of the network onto the flat side (bottom half). All of the progressions then move to another progression on the flat side - either left or right on the graph - and then finally they move back upward across the center again toward the sharp side.

Both *Someone Like You* and *She Used to Be Mine* possess secondary progressions within their respective choruses. *Someone Like You* moves from the original I-iii-vi-IV to a I-V-vi-IV in the chorus. This progression is identical to the primary progression of *She Used to Be Mine*. The chorus of *She Used to Be Mine*, on the other hand, moves briefly to a I-III-vi-IV, which happens to be the primary progression of *I'm Not the Only One*. Figure 39 shows these alterations, and their further connections to the other aforementioned progressions.

In the Adele example, the change between progressions is due to the change between iii and V. The I-V ($Q^{\sharp 1}$) progression is extremely strong, and the I-iii ($M^{\sharp 1+}$) is weaker, yet similar ($Q^{\sharp 1} \leftarrow$

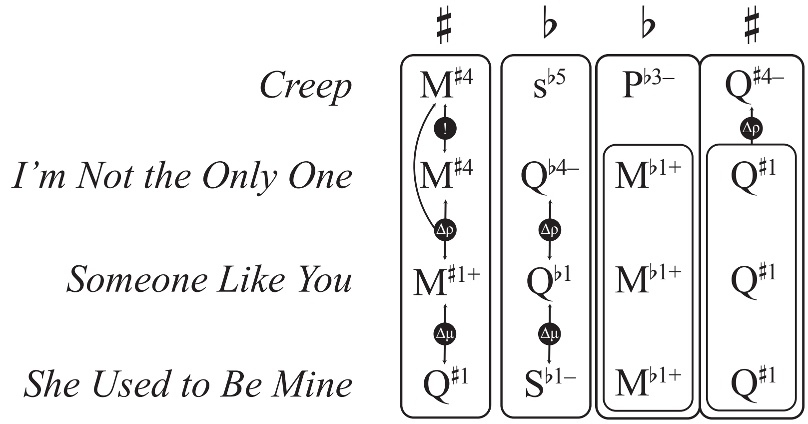


Figure 38: A network showing exactly how the four progressions are related. The ! shows that the progressions are identical.

$\Delta\mu \rightarrow$ M$^{\sharp1+}$). In the M$^{\sharp1+}$, the root motion differs from the harmonic motion: $\Delta\rho = \sharp4$, $\Delta\mu = \sharp1$. The Q$^{\sharp1}$ holds an agreement between root motion and harmonic motion, where both parameters move $\sharp1$. This agreement further strengthens the progression as a true representation of the $\sharp1$ harmonic move.

She Used to be Mine exchanges the I-V (Q$^{\sharp1}$) for a I-III (M$^{\sharp4}$). This also adds strength to the overall progression in the form of a higher magnitude of harmonic motion: $\sharp4$ versus $\sharp1$. In both examples, the reason for switching chord progressions is to enhance the most pivotal moments of each song. The interaction and magnitude of root motion and harmonic motion offer clear candidates for such exchanges to be made that in turn provide *similar* harmonic trajectories between the primary and secondary progressions in each.

5 Harmonic Contour and Schemata

The primary objective in analyzing progressions and their movement through harmonic space is to then relate such progressions to other similar progressions. There are two common schemata that are ubiquitous within harmonic progressions: *Compounding* and *Digression-Regression*. Compounding progressions occur when multiple adjacent chords move in one direction around the circle of fifths, thus emphasizing their direction as a salient parameter of the harmony. Harmonic compounding can exist at any magnitude, but the higher the magnitude, the more salient the compounding becomes. Harmonic compounding can be notated as [/] for sharpward and [\] for flatward. The magnitude of compounding can be described as a fraction. Earlier, Figure 18 showed how the progression from *Creep* compounds flatward [\] at a rate of $\frac{\flat8}{2}$ and sharpward [/] at the same rate: $\frac{\sharp8}{2}$.

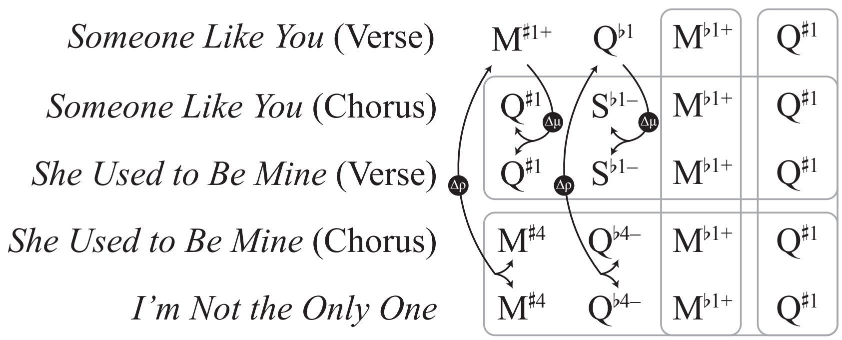


Figure 39: A network showing the secondary progressions in *Someone Like You* and *She Used to Be Mine*, and how they compare to their respective primary progressions, as well as other similar progressions.

The most salient of the Digression-Regression schemata involve a two-chord progression that moves in one direction by a certain amount, followed by the next two-chord progression that moves the opposite direction by the same amount. There are two variants of this schemata: Prime Digression-Regression (∧ or ∨), in which the first chord and the last chord are identical, and Relative Digression-Regression (∴ or ∵), where the first and last chords are relative to one another, thus taking up the same span on the circle of fifths. Prime Digression-Regression is the more salient of the two, because of the identical nature of the first and last chords. Relative Digression-Regression happens more often, but is still a worthwhile schema to take note of. The analytical efficacy of these schemata comes from their specific use in chromatic progressions, rather than diatonic progressions.

There are four other variants of the Digression-Regression schemata that are ubiquitous in many types of harmonic progressions. In all four variants, the schema follows a chromatic two-chord progression that moves a certain distance that is greater than, or equal to 3 fifth-steps, followed by another two-chord progression that moves either one step further or one step closer in the opposite direction. For any major or minor triad, $\sigma = 4$. If one were to project a local diatonic span ($\sigma = 6$) on top of one of these triads, it would be positioned one fifth-step in either direction of the triad. In a three-chord progression that follows one of these schemata, the first chord sets a local diatonic span, and the second chord - because it must possess a ΔT of 3 fifth-steps or higher - moves chromatically outside of the span. If the third chord moves in the opposite direction at a distance that is either one step further or closer than the first progression, it will land inside the diatonic span of the first chord. This binds the first and third chord together due to their diatonic proximity. Figure 40 shows examples all of the possible harmonic schemata. The chords in the figure that are made up of circles indicate that they can be any potential chord, and the schema

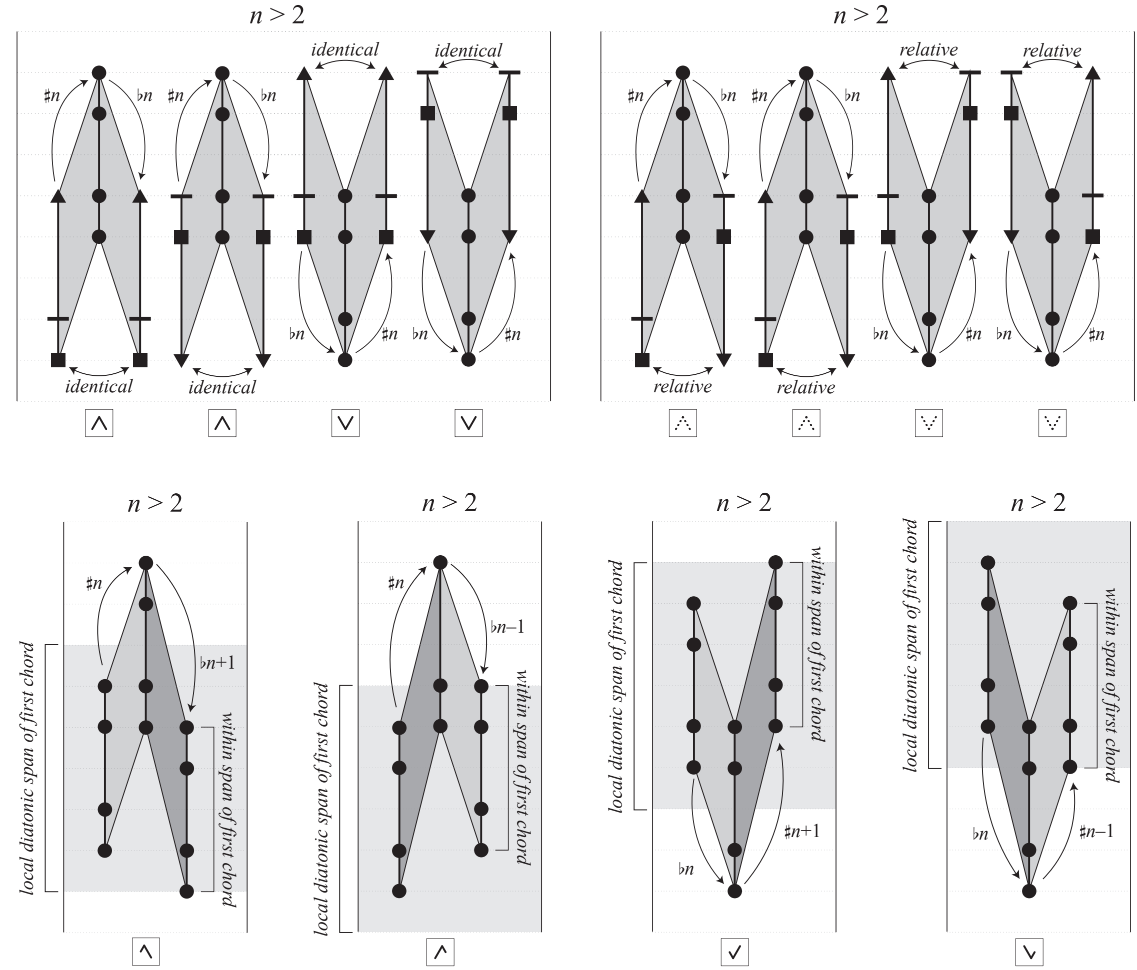


Figure 40: Formulaic examples of different harmonic schemata. Chords with circles indicate that the chord can be of any quality.

will still apply. Below is a table of definitions for each schema.

- ∧ Sharpward digression, equally flatward regression to same chord.
- ∨ Flatward digression, equally sharpward regression to same chord.
- ∴ Sharpward digression, equally flatward regression to different chord.
- ∵ Sharpward digression, equally flatward regression to different chord.
- ^ Sharpward digression by n distance, flatward regression by *n–1*.

- $\boxed{\vee}$ Flatward digression by *n* distance, sharpward regression by *n–1*.
- $\boxed{\wedge}$ Sharpward digression by *n* distance, flatward regression by *n+1*.
- $\boxed{\checkmark}$ Flatward digression by *n* distance, sharpward regression by *n+1*.

Analyzing these schemata makes it possible to compare progressions that have been composed across many centuries of music history. As we have seen when comparing the variations of the I-iii-vi-IV progression, all of the progressions - with the exception of *Creep* - begin by moving sharpward, and then immediately move equally flatward: $M^{\sharp 1+}\rightarrow Q^{\flat 1}$ and $M^{\sharp 4}\rightarrow Q^{\flat 4-}$. These progressions exemplify the $\boxed{\therefore}$ schema. *Creep* differs from this schema, as the move from I-III - $M^{\sharp 4}$ - is followed by a III - IV ($s^{\flat 5}$). The I-III-IV from *Creep* looks similar to $\boxed{\therefore}$ schema presented in the other progressions, but the difference is that while *Creep's* I-III creates a sharpward digression of $M^{\sharp 4}$, the regression of $s^{\flat 5}$ moves the harmony one step further flatward than it did sharpward. This schema of $\boxed{\wedge}$ can be seen as *similar* to the $\boxed{\bigwedge}$ and $\boxed{\therefore}$ schemata.

The Digression-Regression schemata explain how many progressions tend to have a gravitational pull towards some type of center (typically a diatonic span). Oftentimes when a chromatic progression occurs, it - by definition - pushes the harmony outside of the diatonic span. This is commonly followed by another progression that pulls the harmony back to the diatonic span. This contrasts the Compounding schemata, in which multiple progressions move in the same direction, unimpeded by the gravity of the harmonic span.

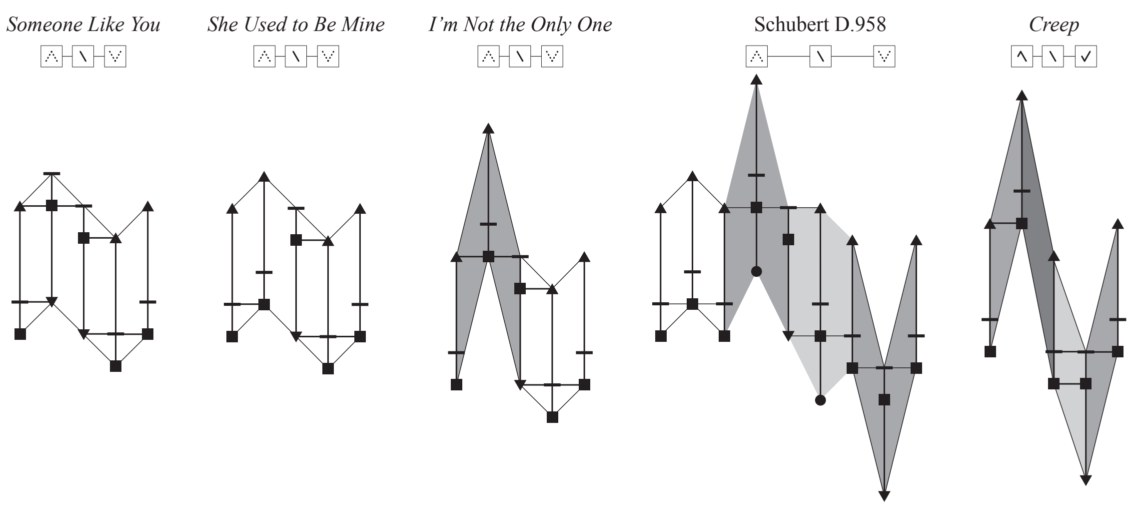


Figure 41: The similarities in harmonic schemata between five unique progressions.

Figure 41 shows the previously analyzed progressions and their respective schemata. *Someone Like You*, *She Used to Be Mine*, and *I'm Not the Only One* all possess the same schemata: $\boxed{\therefore}$

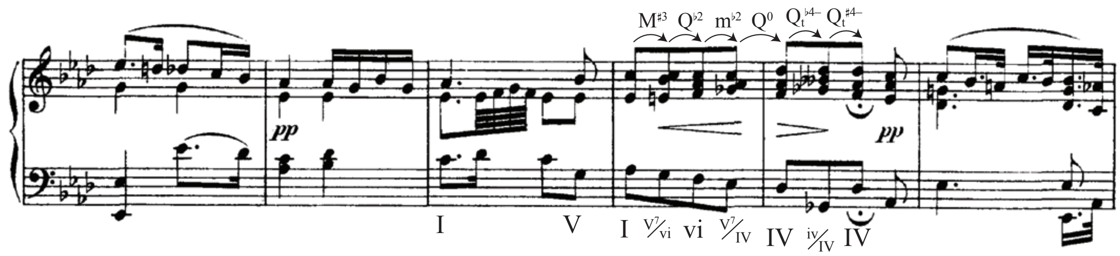


Figure 42: An excerpt from Schubert's *Piano Sonata in C Minor* (D. 958).

→[\] →[∵]. The next progression in the figure is an excerpt of a phrasal ending from Schubert's C Minor Piano Sonata (D. 958). This excerpt *also* possesses a [∴] schema followed by a flatward descent - [\] - and a [∵] schema. The right-most progression - *Creep* - possesses similar schemata to the other examples, but the harmonic "backtracking" is unequal: [∧] →[\] →[✓]. As is similar to the previously defined levels of harmonic analysis, the higher the amount of chromaticism within these schemata, the more strong the comparative analysis will be. For example, the Schubert example is more similar to *I'm Not the Only One* and *Creep* as all three progressions are chromatic. *Someone Like You* and *She Used to Be Mine* are obviously quite similar - and are also similar to *I'm Not the Only One* as well - but their level of schematic similarity is lesser when compared to the Schubert and *Creep* because of the difference in magnitudes of harmonic distance between the examples. Analyzing the general contours between multiple progressions shows a commonality in how harmonic form is composed. This commonality transcends time, making it possible to compare progressions - as in this example - written in 1828 and progressions written almost two-hundred years later.

The progressions of *Someone Like You*, *She Used to Be Mine*, *I'm Not the Only One*, and *Creep* have been analyzed extensively thus far. The reason for choosing these progressions is that they are quite simple, and straightforward. They are textbook candidates for such an analysis of harmonic motion, and a cross-comparison between similar progressions. These progressions provide a concise way to show all of the properties that are at play in an analysis of motion through harmonic space. The following examples throughout the rest of the text will possess much higher levels of complexity, but the salient properties of harmonic motion that have already been discussed remain the same. The interaction between $\Delta\mu$, $\Delta\varphi$, and $\Delta\rho$ provide a rich foundation for extrapolating many unique properties of harmonic motion. These properties, at the most global level, can be used to compare harmonic progressions written in completely different time periods and cultural contexts. A through-line of characteristic harmonic motion can then be drawn, which could lead to further analyses of sonic impetus, salient progressions, and compositional idioms that last through multiple eras of musical creation.

6 Non-Triadic Chords

Jason Yust points out that "there is an analytical and theoretical need for a harmonic space that preserves the common-tone logic of the *Tonnetz* while expanding its range of musical objects beyond major and minor triads"[16]. In progressions that include only major and minor triads, the interaction between $\Delta\mu$ and $\Delta\varphi$ was simple and predictable. The primary reason being that major and minor triads possess identical spans of four spaces on the circle of fifths. When considering chords of higher cardinality such as seventh chords (and the myriad extended chords within the language of Jazz harmony), the respective spans become much more varied. This is turn causes a more unique and unpredictable interaction between $\Delta\mu$ and $\Delta\varphi$.

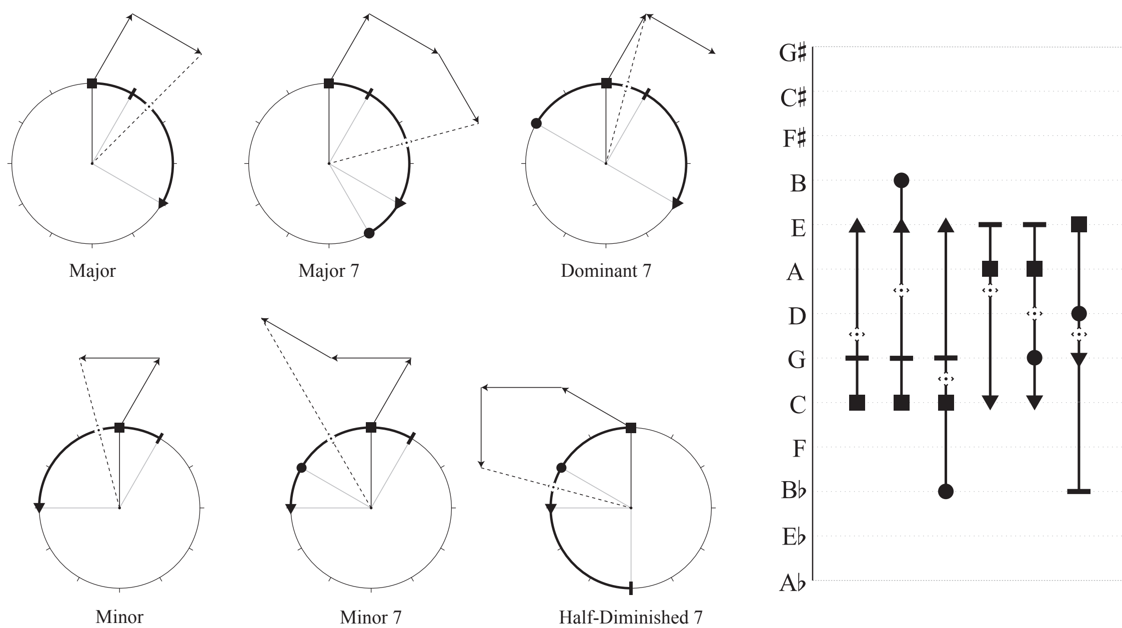


Figure 43: Circular means φ of major and minor triads compared to common seventh chords (excluding the fully diminished seventh chord).

Figure 43 shows how using the φ measurement provides a unique center point for relatively similar chords, and accurately describes the effect that an added seventh has on a triadic major or minor core. The major seventh added to a major triad sharpens the φ from its triadic φ value, and the minor seventh added to a major triad flattens the φ. Hermeneutically speaking, the major seventh brightens the major triad, pushing it sharpward, while the minor seventh darkens the major triad, pulling it flatward. Similarly, the addition of a minor seventh upon the minor triad flattens φ

16. Yust, "Schubert's Harmonic Language and Fourier Phase Space," 127.

slightly from its triadic φ value. The major and minor seventh are the only common chords in which $\varphi = \mu$, as their members are symmetrically balanced around the true center. The half-diminished seventh chord has a large span of 6 - identical to the diminished triad - but the addition of the seventh pulls φ sharpward. The increased variation of both μ and φ in seventh chords causes unique interactions between their two measurements. Therefore, the convenient ΔT measurement does not apply to progressions containing seventh chords, or other chords of higher cardinality. Because of this, the more precise $\Delta\varphi$ will be observed primarily, but with the continued notion that μ and φ affect one another.

7 Conclusion

The proposed analytical model introduces a comprehensive and nuanced method for qualifying the relationships between adjacent chords with respect to their positions in harmonic space. We have explored relationships between chords using extant frameworks of Roman numeral analysis, transformation systems, and voice leading measurements, and have identified that currently, the primary way in which the motion between chords is analyzed is through voice leading. This new model additionally looks at chords through the concept of harmonic distance. Three salient properties of harmonic motion were then accounted for - $\Delta\mu$, $\Delta\varphi$, and $\Delta\rho$ - creating a model for measuring the harmonic distance of many types of harmonic progressions. Observing how these three salient parameters interact with one another creates a meaningful analysis of harmonic progressions that unlocks both interesting features of such progressions that were otherwise unseen, as well as a way to visualize and qualify the most sonically peculiar parts of the progressions. The model becomes even more useful when examining similarities between harmonic gestures between a wide gamut of musical examples.

The relationship between voice leading proximity and harmonic proximity provides an interesting result in that the two are roughly inversionally related; when a progression possesses a high level of voice leading distance, it typically is rather close in harmonic space, and vice versa. This observation - and the fact that it is not a perfect inversional relationship - shows that an analytical model for harmonic proximity can unveil more information about what is truly happening from one chord to another.

By relating harmonic progression across genres and eras, the model reveals underlying similarities and innovations between harmonic structures that were composed across a broad span of music history. This universal applicability makes it indispensable for theorists, musicologists, and educators aiming to connect diverse musical traditions.

Ultimately, by attempting to qualify the harmonic properties of chordal motion first and foremost, this approach offers a key to unlocking rich, previously obscured characteristics of harmony, enhancing both academic discourse and practical analysis. The Harmonic Proximity Graph encourages a rethinking of harmony as not a fixed set of rules, but as an evolving, single language – a concept that is vital as musicians navigate the expanding boundaries of musical expression in the 21st century.

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